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ABSTRACT

This document is a compilation of abstracts of 28 research papers presented at the 52nd Annual Meeting of the National Council of Teachers of Mathematics. The major portion of the reports concerned methods of instruction; nine were related to college mathematics, ten on secondary school mathematics, and two at the elementary school mathematics level. Five reports involved research on learning theories and two dealt with teacher education. (JP)

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MATHEMATICS EDUCATION REPORTS

RESEARCH REPORTING SECTIONS

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

52nd ANNUAL MEETING

Atlantic City, New Jersey
April 17-20, 1974

ERIC Information Analysis Center for
Science, Mathematics, and Environmental Education
400 Lincoln Tower
The Ohio State University
Columbus, Ohio 43210

Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the editor.

FOREWORD

The ERIC Information Analysis Center for Science, Mathematics, and Environmental Education is pleased to make available this compilation of abstracts of the research papers to be presented at the 52nd annual meeting of the National Council of Teachers of Mathematics. Selection of papers was directed by Professor James Wilson of The University of Georgia with cooperation of the AERA Special Interest Group for Research in Mathematics Education. Minor editing has been done by the ERIC staff to provide a general format for the papers. Many of the papers that are abstracted here will be published in journals or be made available through the ERIC system. These will be announced in *Research in Education* and in the journal *Investigations in Mathematics Education*.

April, 1974

Jon L. Higgins
Editor

Sponsored by the Educational Resources Information Center of the National Institute of Education and the Center for Science, Mathematics, and Environmental Education, The Ohio State University in cooperation with the AERA Special Interest Group for Research in Mathematics Education.

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Thursday, April 18, 1974
10:30 - 11:30 A.M.

Pennsylvania Room B(S)
Jerry P. Becker, Presider

RESEARCH REPORTING SECTION I

Reports: THE INTUITIVE CONCEPT OF A LIMIT POSSESSED BY PRE-CALCULUS COLLEGE STUDENTS AND ITS RELATIONSHIP WITH THEIR LATER ACHIEVEMENT IN CALCULUS

DOROTHY T. COON, Wheeling College, Wheeling, West Virginia

THE EFFECTS OF VARIATIONS IN PACE OF INSTRUCTION AND IN TEXTBOOK STYLE ON THE LEARNING OF ELEMENTARY ALGEBRA BY LOW ACHIEVERS

ALBERT HENRY MAUTHE, JR., Norristown Area School District, Norristown, Pennsylvania

THE EFFECTS OF QUESTIONS OF DIFFERING COGNITIVE LEVELS INTERSPERSED IN MATHEMATICS TEXT MATERIAL

KENNETH W. WUNDERLICH, The University of Texas at San Antonio

L. RAY CARRY, The University of Texas at Austin, Austin, Texas

THE EFFECTS OF TRAINING IN HEURISTIC METHODS ON THE ABILITY TO WRITE PROOFS IN NUMBER THEORY

DOROTHY JEANNE GOLDBERG, Newark State College, Newark, New Jersey

THE INTUITIVE CONCEPT OF LIMIT POSSESSED BY PRE-CALCULUS COLLEGE STUDENTS AND ITS RELATIONSHIP WITH THEIR LATER ACHIEVEMENT IN CALCULUS

Dorothy T. Coon
Wheeling College

Purpose

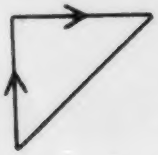
This study examined the pre-calculus college student's insights into the concept of limit. Information was sought concerning (1) the pre-calculus college student's intuitive concept of limit, (2) the relationship between the intuitive concept of limit and later achievement in calculus, and (3) the difference between the intuitive concept of limit possessed by male and female students.

Research Design and Procedure

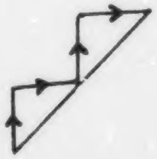
The population studied was those students at The Ohio State University who enrolled in Algebra and Trigonometry in the Fall quarter of 1971 and were planning to enroll in Calculus and Analytic Geometry the following quarter. The experimental sample was selected from volunteers from this population. Twenty-one male and eighteen female students actually participated.

Six tasks originally used by Stanley F. Taback were administered individually to the subjects. One involved the concept of neighborhood. In each of the others, a function was defined and the student was required to demonstrate his understanding of the function, its convergence or divergence, and the limit, if any. A brief description of the tasks follows:

1. The first four squares of a sequence of diverging squares are presented to the subject. Each square, after the first, passes through the vertices of the preceding one. The questioning relates to the unbounded increase in the areas of the squares.
2. Two rays, each representing a street, form an angle whose measure is 45 degrees. The vertex of the angle represents a man's home. The man starts at a point on one street and walks back and forth between the two streets, each time in a direction perpendicular to the street toward which he is heading. The questioning relates to the convergence of the man's paths to the point representing his home.
3. Point A on a given line and a separate line segment one inch long are specified. The questioning relates to the subject's ability to identify the open interval on the line and the open disk in the plane centered at A and with radius one inch given the defining properties of each.
4. A rabbit hops halfway from one end A of a line segment AB toward the other end B, then hops halfway again from where he is toward point B. The rabbit continues to hop, following the same pattern of hopping halfway from wherever he is toward point B. The questioning relates to the convergence of the hops to the point B.
5. An ant crawls up a given triangularly shaped step, first traveling vertically and then horizontally, as indicated by the arrows.



The step is replaced by two smaller steps. The path traveled by the ant is again indicated by the arrows.



Then each of the two steps is broken up and similarly replaced by two steps. This process continues indefinitely. The questioning relates to the constancy of the distance traveled by the ant and the convergence of the area underneath the steps to zero.

6. Two rabbits are involved in this task. One rabbit hops along a line segment AB exactly as the "Halfway Rabbit" previously defined. The other rabbit, a "Probabilistic Rabbit," hops along a line segment CD congruent to AB as follows: Every time the Halfway Rabbit takes a hop, the Probabilistic Rabbit flips a coin. If he gets a head, he covers as much distance as the Halfway Rabbit just covered on his last hop; if he gets a tail, he remains where he is. The questioning relates to the convergence of the distances covered by the Probabilistic Rabbit in the absence of a specific limit point.

Each interview was tape-recorded and subsequently rated in terms of a predetermined rating scheme. An overall composite score and function, convergence, and limits subscores were computed for each student.

Findings

With regard to the pre-calculus college student's intuitive concept of limit, it was found that, of the subjects involved:

1. A minimum of seventy-nine percent clearly understood the functions defined.
2. Eighty-seven percent clearly described the neighborhoods involved in task 3.
3. A minimum of seventy-nine percent understood the convergence or divergence of the functions.
4. A minimum of forty-nine percent correctly identified and understood the limits involved.
5. Only forty-one percent understood the final task which involved concepts of probability and least upper bound.

Therefore, less than half of the pre-calculus college students represented in this study could demonstrate that they clearly understood all of the phases of the concept of limit as defined by the tasks employed.

Achievement data in calculus for each subject were obtained in the quarter immediately following the interviews. These data consisted of each subject's score on selected questions related to limits from the first examination in calculus, and each student's final grade in the calculus course. Pearson product-moment coefficients of correlation were computed between the composite scores and (1) possession or non-possession of a high school calculus background, (2) standard scores on the mathematics section of the ACT entrance test, (3) scores on the selected items from the first examination in calculus, and (4) final grades in calculus; and between the limits subscores and the final grades in calculus. None of the computed correlations were significant at the .05 level.

T-tests were used to test for differences in scores between male and female subjects. Males had significantly better limits subscores than females (.05) level). No significant differences were found for the function or convergence subscores, nor for the composite scores.

Interpretations

This study adds to a growing body of evidence which seems to refute Piaget's conclusion that formal patterns of reasoning are well established in nearly all persons over the age of twelve to fourteen. The pre-calculus college students in this study performed well on questions about functions, convergence, and neighborhoods involving concrete or pictorial representations. However, performance on the more abstract questions about the limits of the functions was markedly different. Answering these questions required the student to ignore pencil and paper limitations and to think beyond specific instances. Implications of continuing processes had to be considered which required abstract reasoning. Thus it appears perilous to assume that beginning calculus students can reason abstractly in finding limits of sequences. Furthermore, since neither the ACT Mathematics scores nor the grades in the calculus course correlated significantly with the composite scores, it may be concluded that the intuitive knowledge tested by the limits tasks differed from the factors tested by the calculus examinations and the ACT Mathematics test; i.e., success in beginning calculus does not appear to depend on the student's intuitive concept of limit.

THE EFFECTS OF VARIATIONS IN PACE OF INSTRUCTION AND IN TEXTBOOK STYLE ON THE LEARNING OF ELEMENTARY ALGEBRA BY LOW ACHIEVERS

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Norristown, Pennsylvania

Purpose

The purpose of the study was to determine to what extent low-achieving students benefit in learning algebraic concepts by a reduced pace of instruction, by a modification of the algebra textbook especially written for low achievers, or by a combination of a reduced pace and a modified textbook.

Research Design and Procedure

The 120 subjects were of average intelligence and had been recommended for a two-year sequence of elementary algebra because of their low achievement in mathematics. They were enrolled in the second year of such a program. The students were assigned at random to four classes that were as similar as possible in previous achievement, as measured by a standardized algebra test.

Two paces of instruction were used, one was approximately the pace for the usual one-year algebra course, and the other was approximately half as fast. Two School Mathematics Study Group textbooks having parallel treatments of the same topics were used; one was written for approximately the top third of the school population, and the other was a modification written especially for lower achievers. Each of the classes studied each of four units of material under a different combination of textbook and pace.

At the conclusion of each unit, a unit test was administered. Another test, administered toward the year's end, was considered as four delayed posttests, each consisting of test items for one unit.

For purposes of analyses, the students were divided into achievement levels, with those repeating the course placed into one level. The remainder were divided into three levels on the basis of the standardized algebra test given prior to the experiment. A three-way analysis of variance (two textbooks by two instructional paces by four achievement levels) was run.

Findings

On the immediate posttests, there were significant differences for Unit I ($p < .05$) and for Unit IV ($p < .001$), in favor of the modified textbook groups, but not for the other units. On the delayed posttests, there were no significant differences between textbooks. On the delayed posttests for Units I, II, and III, there were significant level by textbook interactions ($p < .05$). For Units I and II, but not for Unit III, the students at the middle and lower levels seemed to profit relatively more from the modified textbook than did the students at the higher level.

On the immediate posttests, there were significant differences for Unit III ($p < .001$) in favor of the slower-paced group, for Unit IV ($p < .01$) in favor of the faster-paced group, but not for the other units. On the delayed posttests, there were significant differences for Unit I

($p < .05$) and for Unit III ($p < .001$) in favor of the slower-paced groups, but not for the other units. On the immediate posttest for Unit IV, there was a significant textbook by pace interaction ($p < .05$): the advantage of the modified textbook group over the regular textbook group was greater at the faster pace than at the slower pace.

The classes studying from the modified textbook at the slower pace had the highest means on each of the Unit I and III posttests, the second highest means on the Unit IV posttests, and the lowest and the second lowest means on the Unit II posttests.

A test of ranks of class means for the eight unit posttests indicated that the rank totals differed significantly ($.01 < p < .02$). Some of the conflicting results for pacing may therefore have been attributable to differences between the classes. The design did not permit the clear separation of effects. The joint variation of two instructional characteristics, however, showed promise as a technique for studying mathematics learning.

THE EFFECTS OF QUESTIONS OF DIFFERING COGNITIVE LEVELS INTERSPERSED IN MATHEMATICS TEXT MATERIAL

Kenneth W. Wunderlich
The University of Texas at San Antonio

L. Ray Carry
The University of Texas at Austin

Purpose

This investigation sought to determine whether interspersing questions in mathematics text after segments of content to which the questions related effected learning. The study also sought to determine if different cognitive levels of questioning during reading differentially effected learning.

Rationale

E. Z. Rothkopf of Bell Laboratories developed a model for mathemagenic behavior, behavior giving rise to learning. The model is an adaptation of a model for learning paired-associates by means of prompting or confirmation to learning from a written sentence (Rothkopf, 1963). The model suggests that what is learned when reading a sentence is a result of the articulated response to the written words which serve as the stimuli. Intonation patterns and previous associations made with the words serve to condition the stimuli and shape the articulated response. It is the articulated responses and the stimuli that are consequences of these articulated responses that result in learning.

Research utilizing questions as a means of shaping the inspection behavior of the reader is an outgrowth of this model. The questions tend to predict what words will be critical in the formation of the effective stimulus resulting from an articulated response and how the conditional relationship between the stimulus consequences and the articulated response is formed. The use of questions interspersed in reading passages has been found to shape what the reader learns (Rothkopf, 1966; Rothkopf and Bisbicos, 1967; Frase, 1967). The position of the question relevant to the subject matter to which it refers effects the function of the question. Questions placed after the subject matter to which they pertain have been found to improve retention of the specific content of the particular question as well as shape the reader's inspection patterns so as to improve learning of similar subject matter. Such subject matter had not been specifically questioned and was not directly related to the subject matter which was questioned. An optimal passage length between questions has been found to be approximately 20 lines for prose material (Frase, 1968).

The nature of the questions in early studies was primarily such that they would be classified as cognitively simple, although no effort was apparently made to distinguish between cognitive levels. Later studies have been concerned with the effect of specific and general questions. The nature of such questions has little effect upon the functions such questions have been found to perform when placed after the relevant passage. However, general questions impeded learning when placed before the passage (Frase, 1970). Hunkins (1968, 1[illegible] found that questioning students at the evaluative level as opposed to the knowledge level (Bloom, 1956) resulted in no difference in performance on a criterion test composed of knowledge level questions. The performance of the group receiving evaluative questions was significantly better than that of the group receiving knowledge level questions on a criterion test composed of evaluative questions.

Mathematics content had never been used in such studies. Rothkopf (1969) invited the extension to new subject matter and more involved thought processes. Frase (1970) suggested that questions adhering to a taxonomic classification be employed to study a variety of behaviors if precise classifications could be found. Hence, this study was undertaken. As a result of the previous research, it was hypothesized that experiencing questions of any of three cognitive categories used, during reading would result in more learning than would the control condition. It was also hypothesized that inspection behaviors induced by questions of a higher congitive level would result in as much learning of content, conducive to questioning at a lower congitive level, as would questioning at the lower cognitive level.

Research Design and Procedure

To test these hypotheses, a discussion of mathematical functions was divided into 14 passages each approximately 20 lines in length. Two multiple choice questions for each of three cognitive categories were designed for each of the 14 passages. Definitions by Gall et. al. (1969) were used to develop questions of the knowledge, comprehension, and application or analysis level. Questions were rated by a panel of 3 judges and unanimous agreement was attained by the panel and the investigator prior to inclusion of a question in the experimental materials. The two questions of each level were randomly assigned to serve either as an item of the criterion test or as a question interspersed in the reading passage. That there was no direct transfer from the set of interspersed questions to items of the criterion test was determined by a pilot study, making tenable the investigation of general facilitative effects of the questioning.

Subjects were randomly assigned to read one of five treatment booklets. Four treatment groups received booklets containing one question after each of the 14 passages. Three of these groups experienced questions during reading which were from only one of the three cognitive categories. The fourth group experienced questions from each of the three categories with questions becoming cognitively more complex as the discussion developed. The fifth treatment group received no questions but had instructions to carefully read the passage, paying attention to details.

Subjects were 163 geometry students from a high school serving a city of 20,000. Three class periods, each 55 minutes long, were utilized. The instructional booklets were read and the criterion test administered on the first two days. Two weeks later, a 30 item retention test was administered to determine retention of general facilitative and direct learning effects of the questioning.

Findings

Data was analyzed using analysis of variance. Specific comparisons to test for the heirarchical effects hypothesized were conducted using the t-test for the difference between means. Results of the criterion test indicated that questioning of any cognitive level did not facilitate learning when compared with the group which only read the passage. When the three subscales were considered, the group experiencing knowledge level questions scored significantly higher than the group experiencing questions of all three levels on the knowledge subscale. The group experiencing knowledge level questions performed as well as the other groups experiencing questions on the comprehension and application or analysis subscales.

Results of the retention test indicated that there were no differences among the treatment groups. When items experienced only on the criterion test and subscales of these items according to cognitive level were considered there were no differences. When the items of the retention test which had also served as questions interspersed in the treatment booklets were considered, the group which had experienced knowledge level questions scored significantly higher than each of the other treatment groups on the knowledge subscale.

Reliability of the criterion and retention test was measured by computation of an alpha coefficient. An alpha coefficient of .63 was computed for the criterion test and of .64 for the retention test.

Interpretations

The results of this study indicate that interspersed questions did not facilitate learning when compared with a group reading the passage without questions. The heirarchical inspection behaviors hypothesized to result from experiencing questions of higher cognitive order also did not occur. The amount of time spent reading the treatment materials as reflected by the number of subjects failing to respond to interspersed questions and the magnitude of the means which favored the group experiencing knowledge level questions suggests an unfamiliarity with questions at higher cognitive levels. The experimental arrangements may not have allowed appropriate processing of questions at the higher cognitive levels.

Results of this study do not suggest, finally, that such questioning does not facilitate learning of mathematics material but instead suggest a need for more refined questioning by use of subcategories of the categories employed and test revision to insure greater statistical reliability.

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THE EFFECTS OF TRAINING IN HEURISTIC METHODS ON THE ABILITY TO WRITE PROOFS IN NUMBER THEORY

Dorothy Jeanne Goldberg
Newark State College

Purpose

The purpose of this study was to investigate the effects of instruction in heuristics on the ability of college students not majoring in mathematics to construct proofs in number theory.

Rationale

To understand what mathematics is, sooner or later students must become familiar with the nature of proof. To really understand proof, students should learn to write proofs. But it has long been recognized that learning proof construction is not easy. Polya, the foremost authority on heuristic, formulated a set of common sense questions and suggestions for constructing proofs. Some simple heuristics that Polya identified as useful for solving a problem "to prove" are: What is the hypothesis? What is the conclusion? Can you introduce suitable notation? Look at the conclusion. Try to think of a familiar theorem having the same or a similar conclusion. Look at the hypothesis. Can you derive something useful from the hypothesis? Can you bring hypothesis and conclusion nearer to each other? Did you use the whole hypothesis? Should you go back to definitions? Polya believed that the teacher who used such heuristics as he constructs proofs with his class teaches more than the proof itself; he is teaching a general problem solving method. Eventually, the students will learn how to pose the heuristic questions and suggestions to themselves.

Polya's heuristics for constructing proofs seem eminently suitable as a process for college students to learn to facilitate their efforts in writing proofs. It should be worthwhile to evaluate experimentally the effectiveness of such training. The present study compares several instructional programs in proof construction.

Research Design and Procedure

Two sets of programmed booklets were written by the investigator. In one set of seven booklets, instruction in heuristics for proof was given, along with a sequence of number theory problems. In the other set of seven programmed booklets, the same sequence of problems was presented together with additional problems, but without instruction in heuristics.

Nine number theory classes, with 238 students enrolled in all, participated in the experiment. The classes met twice weekly, seventy-five minutes each meeting, and worked with the programmed booklets seven out of the twelve class meetings of the six-week experimental period.

The nine classes were randomly assigned to one of three different treatments. In the reinforced heuristic treatment, the classes used the programmed booklets that contained instruction in heuristics. During class discussions, the instructor was to reinforce the learning of the heuristics by using them as often as possible. In the heuristic treatment, the classes used the same programmed booklets, but the instructor was not to reinforce the use of heruistics in class. He was merely to demonstrate -- "This is how the proof is done." The classes in the non-heuristic treatment used the programmed booklets that contained no instruction in heruistics. The instructor was not to use heuristics in class, as in the heuristic treatment.

Three instructors each taught two classes, and three instructors each taught one class. One of the classes had only six students enrolled.

Posttests on number theory concepts and on proof construction, an attitude questionnaire on the self-instructional booklets and Covington's Childhood Attitude Inventory for Problem Solving were given after the experimental period. Five weeks later delayed posttests on concepts and on proof construction were given. SAT scores were available for all but five of the students.

Findings

When pairs of classes assigned to different treatments were compared by constructing confidence intervals using the t-statistic ($\alpha < .05$), there was some evidence, but not at a statistically significant level, that reinforced heuristic instruction was the most beneficial of the three instructional methods. The hypothesis of no difference between unreinforced heuristic instruction and non-heuristic instruction was supported.

The SAT-mathematical scores and the posttest scores of all the students were ranked into thirds - high, medium, and low - and a chi-square analysis with four degrees of freedom was used to determine whether treatment was associated with posttest performance at each ability level. For the high ability students, there were significant differences in understanding of concepts, and differences, although not significant, in ability to construct proofs, favoring those students given reinforced instruction in heuristics. High ability students who were given non-heuristic instruction understood concepts significantly better and tended to construct proofs better than their counterparts who received unreinforced heuristic instruction. In this study high ability students had SAT-mathematical scores ranging from 491 to 679. Only 4 per cent of the students had SAT-mathematical scores above 600. Among low and medium ability students, there were no significant differences in understanding of concepts or in ability to construct proofs.

In every treatment group, there was a strong positive relationship between ability to construct proofs and mathematical ability, and between ability to construct proofs and understanding of concepts. Both relationships were strongest for the group given reinforced heuristic instruction.

Of those students who scored in the top third on the posttests, the students who were given reinforced instruction in heuristics used the greatest variety of heuristics, and used them most frequently. The students who were given non-heuristic instruction had a more positive attitude toward the self-instructional booklets and toward the nature of the problem solving process than the students given either reinforced or unreinforced heuristic instruction. The students who were given unreinforced instruction in heuristics had the most negative self-images.

Interpretations

It is sometimes said that students should learn proof by doing many proofs. The results of this study suggest that, at least for some college students not majoring in mathematics who are studying number theory, giving students explicit instruction in some heuristics of proof, together with reinforcement through class discussions, is more effective than merely giving them proofs to do.

Thursday, April 18, 1974
1:30 - 2:30 P.M.

Pennsylvania Room B(S)
F. Joe Crosswhite, Presider

RESEARCH REPORTING SECTION II

Reports: AN INVESTIGATION OF TRANSFER OF LEARNING AS MEDIATED BY THREE INSTRUCTIONAL METHODS OF TEACHING SELECTED MATHEMATICAL GENERALIZATIONS

COY EDWIN MCCLINTOCK, Florida International University, Miami, Florida

THE RELATIVE EFFECTIVENESS OF GUIDED DISCOVERY AND INDIVIDUALIZED INSTRUCTIONAL PACKAGES IN PROMOTING INITIAL LEARNING, TRANSFER, AND RETENTION IN SECOND-YEAR ALGEBRA

CHRISTIAN R. HIRSCH, Western Michigan University, Kalamazoo, Michigan

COMPUTER SIMULATIONS FOR SENSITIZING MATHEMATICS METHODS STUDENTS TO QUESTIONING BEHAVIORS

JANICE L. FLAKE, Eastern Illinois University, Charleston, Illinois

COMPUTER EXTENDED INSTRUCTION IN INTRODUCTORY CALCULUS

MARTIN T. LANG, California Polytechnic State University, San Luis Obispo, California

AN INVESTIGATION OF TRANSFER OF LEARNING AS MEDIATED BY THREE INSTRUCTIONAL METHODS OF TEACHING SELECTED MATHEMATICAL GENERALIZATIONS

Coy Edwin McClintock
Florida International University

Purpose

The purpose of this study was to investigate the way students can be taught generalizations so that the learning will transfer positively to new learning tasks. Three methods of instruction were designed around the investigator's perception of theoretical constructs of Ausubel and Gagne, Glasser, and Bruner. These methods represented points along the expository-discovery continuum. The experiment compared expository, rule and example, and discovery methods of teaching selected mathematical principles on dependent variables of transfer of learning involving representational, contextual, and difficulty level components of the domain of transfer. It was expected that subjects taught by the more discovery oriented method would perform better on tasks most dissimilar to tasks of instruction, while subjects taught by the more expository oriented method would perform better on tasks most similar to tasks of instruction.

Research Design and Procedure

The subjects were 180 students from private, church-related and public schools of urban communities in the southeastern United States. All subjects were enrolled in a high school geometry course at the time of the experiment. They were self-instructed through semi-programmed booklets. The generalizations of instruction involved seven mathematical sequences and included sequences of partial sums derived from the sums, sums of squares, and sums of cubes of the natural numbers. While the principal independent variable was method of instruction, school class and mathematical achievement levels were also used in statistical analyses.

The dependent variables were investigator defined "factors" of transfer of learning. These factors were measured by a transfer test that required the derivation of algebraic expressions for the general terms of sequences. This test, designed by the investigator, was constructed to allow three partitions, each focusing on a particular "factor" of transfer of learning. These partitions were devised to determine differences in mediational effects of the three methods of instruction. They allowed for an examination of the effect of method of learning on transfer to principles drawn from various contexts, with differing presentational forms, and with stratification of difficulty of subtests.

Findings

Subjects of each instructional mode were able to positively transfer learning to new learning tasks. Transfer to tasks more similar to tasks of initial learning was greater than transfer to very dissimilar tasks. Further, students in the highest achievement strata showed the greatest transferability on each of the transfer variables.

In general, this study supports the hypothesis that learning involving mathematical principles can be transferred positively to tasks involving mathematical principles equally well regardless of whether the instruction is based on exposition, rule and example, or discovery approaches. No significant differences were found to suggest that a particular method affects the transfer of learning from a particular set of generalizations to another set of generalizations better than any other method. Further, no mediational advantage was evidenced with respect to either differing presentational forms or various contextual categories. Advantage of the rule and example method over both the expository and the discovery method occurred at the least difficult transfer level; however, no advantage for any method was evident at other difficulty levels.

Interpretations

Based on this study and the assumption that "factors" of the transfer variable would facilitate research related to the learning of rules and principles for improvement of transfer of learning, further research is suggested. For example, a more detailed analysis of the transfer variable and correlated study of the effects of method upon the factors of transfer should be pursued.

THE RELATIVE EFFECTIVENESS OF GUIDED DISCOVERY AND INDIVIDUALIZED INSTRUCTIONAL PACKAGES IN PROMOTING INITIAL LEARNING, TRANSFER, AND RETENTION IN SECOND-YEAR ALGEBRA

Christian R. Hirsch
Western Michigan University

Purpose

The purpose of this study was to examine the relative efficacy of three modes of instruction for promoting initial learning, vertical transfer, lateral transfer, and retention of mathematical concepts and generalizations. The treatments investigated included guided discovery with emphasis on teacher-student development of definitions, mathematical principles and generalizations through dialectical dialogue; individualized instructional packages developed in an expository format; and individualized instructional packages developed in a branching programmed format.

Rationale

It is generally recognized that one of the major pedagogical goals of mathematics instruction should be the active participation of students in the learning process. In reaction to traditional teaching methods and classroom organization which often tend to inhibit active learning on the part of students, it is becoming increasingly popular to organize instruction so that the meeting between mathematics and the student is on an individual basis.

To date a substantial portion of the literature on individualized instructional programs can be classified as theoretical discussions or descriptions of developmental efforts. Those empirical studies reported have been primarily concerned with differences in achievement between individualized classes and conventional classes. The finding of no significant difference in mathematics achievement has been most frequent. Conspicuously lacking are empirical investigations of the effect of individualized modes of instruction upon transfer and retention.

On the other hand, various mathematics educators who worked on curricula during the late fifties and early sixties have urged that it is possible to present the fundamental structure of the discipline in such a way that students can discover much of it for themselves within conventionally organized classrooms. Though considerable research has been conducted to assess the effectiveness of discovery teaching and learning, the results of these studies suggest that the effectiveness of this approach is still an open question. The present study was an attempt to provide empirical data regarding the relative efficacy of the above two alternatives to conventional instruction.

Procedure

The instructional methods, materials, and criterion measures were developed and tested in preliminary studies at University High School, University of Iowa during the spring and fall of 1971. The study itself was carried out in the spring of 1972.

This study assessed the effects of the three treatments in teaching an extended unit on the field of complex numbers to 213 second-year algebra students in six comprehensive high schools in eastern Iowa. In an attempt to eliminate any differential Hawthorne Effect and to help assure the optimal effectiveness of the treatments, schools and treatments were matched so that subjects in each treatment had prior experience in a program which employed a similar mode of instruction.

The complex number system was developed from the viewpoint of extension fields. Some work with the ordered-pair notation was also included. Each of the instructional packages, designed around one of six sub-categories, contained specific behavioral objectives related to the sub-category, instructional materials to reach these objectives, and self-checking exercises to determine if the objectives had been met. The mathematical content and mathematical approach to the content were identical for both sets of packages; they differed only with respect to programming style. Subjects progressed through the instructional packages at their own rates, but were able to complete one package each day. In the guided discovery treatment, definitions were formulated and properties and generalizations were discovered by the students themselves. Careful guidance on the part of the instructors assured that content was consistent with that found in the packages. Moreover, exercises provided subjects in this treatment were identical to those found in the instructional packages.

Outcomes of instruction were measured sequentially by tests of initial learning, vertical transfer, lateral transfer, and six-week retention. Each of the criterion measures was constructed by the investigator specifically for this study. The content validity of each of the four tests and the construct validity of the two transfer tests were judged by a panel of three members of the mathematics education faculty.

The initial learning and retention tests each consisted of 25 multiple choice items. Though content was limited to concepts and generalizations developed during the experiment, several items evaluated more than a single objective as specified in the set of instructional packages.

The vertical transfer test consisted of 11 problems which were more advanced than those with which the subjects were working. Tasks included graphing of a function of a complex variable; lexographic ordering; determining square roots; and formulating and proving generalizations about conjugates.

The lateral transfer test consisted of 7 items which evaluated student ability to generalize what was learned about the field of complex numbers to other mathematical structures possessing similar structural characteristics. Structures investigated were the field $Q(\sqrt{2})$ where Q is the field of rational numbers; the ring of 2 x 2 matrices over the field of real numbers; the group $(Z \times Z \times Z, +)$ where Z is the set of integers and addition is defined component-wise; and the semi-group $(R \times R, \cdot)$ where R is the set of real numbers and multiplication is defined component-wise. With the exception of finding idempotent and nilpotent matrices, problems were of the same level of complexity as those considered during the experimental unit.

Findings

Bivariate analysis of covariance was applied to the data about the three treatments on each of the four criterion measures. Pretest scores on the Cooperative Mathematics Tests Algebra II, Form B and on the ITED Quantitative Thinking subtest were used as covariates. Prior to each analysis, the assumption of homogeneity of regression was tested, and in each case homogeneous regression was clearly tenable. The Scheffé method for post-hoc comparisons was used to test differences between means for individual pairs of treatments.

Significant differences in initial learning, vertical transfer, and lateral transfer were found beyond the .01 level. Pairwise comparisons of the three treatments indicated that the guided discovery treatment group had, in each case, a significantly higher ($p < .01$) adjusted posttest mean than that of either the expository package treatment group or the programmed package treatment group. No significant differences were found between the two individualized modes of instruction.

With respect to six-week retention, the obtained over-all F had a "p" value of .065. Observed differences again favored the guided discovery treatment over either individualized instruction treatment.

Interpretations

The results of this study indicate that guided discovery techniques which emphasize dialectical dialogue as the mode of instruction may be a more viable alternative to conventional mathematics instruction than individualized instructional packages.

THE USE OF INTERACTIVE COMPUTER SIMULATIONS FOR SENSITIZING MATHEMATICS METHODS STUDENTS TO QUESTIONING BEHAVIORS

Janice L. Flake
Eastern Illinois University

Purpose

The purpose of the study was to investigate the feasibility of utilizing interactive computer simulations to sensitize mathematics methods students to various questioning behaviors, where feasibility for the purpose of this study was defined as the capability of designing, developing, and utilizing a computer program which would encourage effective questioning behaviors by the participant.

Research Design and Procedure

The conceptual framework for designing the program used in the study consisted of the following: the participant was to gain deeper insights into types of questions to ask, sequencing of questions, and ways of reacting to student responses by "teaching" a fictitious class through the use of an interactive computer terminal in the PLATO computer system. The participant was to "ask" a question through the terminal, get a response, etc. Through the questions the fictitious students are to have "learned" a principle.

The design of the program, developed for this study, consisted of articulating models for: lesson planning, Henderson's moves and strategies, Polya's approach to problem-solving, and a simplified learning theory. Also included were options for various questioning behaviors such as: distribution of questions among class members, calling on a student first or asking a question first, wait-time, reacting to a student response and/or student question, redirecting, and probing. Feedback was given in the program in the following forms: fictitious student responses, fictitious student test results, and feedback on questioning behavior.

Procedures for doing the study consisted of conceptualizing the classroom discourse to be simulated, designing a program, and developing a program. During the developing stages a total of three classes, or a total of 64 students, were used. Needed adjustments in the program were made. The program was utilized in conjunction with class discussion, in which the same behaviors were discussed and exemplified. The behavior change of the last group of participants, a total of 25 students, was assessed.

Findings

Participants reacted most favorably towards the program. They liked the involvement that the program gave them. They also felt that the program gave them the opportunity to make mistakes and to be evaluated immediately without harming real students. Of the final class assessed, 22 out of 25 demonstrated an increase of going beyond first responses of a student, 19 out of 25 increased their behavior of modeling the prescribed problem-solving strategy, and 24 out of 25 demonstrated a positive attitude towards the over-all unit.

Interpretations

It was felt that the use of such a computer assisted instruction component is indeed feasible. Furthermore, in designing such a program, much care should be used to work from models that have been abstracted from the real world. Also much care should be used in the kinds and types of feedback that are given, the more immediate the feedback, the more attention the participants paid to it.

The significance of the research has been primarily that it has created a model for such a computer assisted instruction component for a teacher education program, which can yield effective results.

COMPUTER EXTENDED INSTRUCTION IN INTRODUCTORY CALCULUS

Martin T. Lang
California Polytechnic State University

Purpose

The effective utilization of the computer as an integral tool of mathematics education is a concern which has been voiced with increasing tenor in recent years. This research study concerned itself with the development, implementation and evaluation of a particular mode of Computer Extended Instruction (CEI) in a college introductory calculus course. In this mode of CEI, students were provided certain prepared computer programs that were intended to facilitate the learning of some of the basic concepts of calculus. The programs, which were developed by the author, dealt with the topics of functions, graphs, limits and derivatives, and were designed for use by students with little or no knowledge of computer programming. The student utilization of the programs was accomplished on a large computer system (CDC 6600/6400) operating in a multi-processing mode. The calculus programs were stored on disk files and could be accessed from any one of five remote batch-processing centers by submitting a few control cards. Output was received from high-speed printers at the same locations with a turn-around time of from 5 to 10 minutes.

Research Design and Procedure

The sample for this experiment was comprised of eight sections of introductory calculus at The University of Texas at Austin in the Fall semester of 1972. The sections were randomly partitioned into two groups, one designated CEI and the other Non-CEI. While conventional instruction with no change in content prevailed in all classes, supplementary homework assignments requiring the use of the special computer programs were given to the CEI group. The Non-CEI group also received supplementary assignments dealing with the same topics but requiring no computer use.

The experimental design was the pretest-posttest control group design. Two tests in precalculus mathematics, the Algebra III and Trigonometry of the Cooperative Mathematics Tests (ETS), were used as covariates. An author-produced Calculus-Concepts Test and the Calculus Test (Part I) of the Cooperative Mathematics Tests served as two separate criterion measures to determine the effectiveness of the computer augmentation.

Findings

The primary question which was tested was: Does the use of CEI in an introductory calculus class have a positive effect on achievement and concept attainment? The statistical analysis revealed that an affirmative answer seems justified when the focus is on concept attainment. When considering those students who actively participated in the experiment, a nearly significant ($\alpha = .059$) difference in adjusted means was observed on the Calculus-Concepts Test favoring the CEI group over the Non-CEI group. For the subsample consisting of male students only, this difference was clearly significant ($\alpha = .016$), again favoring the CEI group. No significant differences were observed between the groups when acheivement was measured by a test of general knowledge (the Calculus Cooperative Mathematics Test).

Questions regarding the interaction of the treatment with levels of prior achievement in precalculus mathematics and with sex classification were also investigated. The results indicated that when the criterion is concept attainment there is no reason to suspect any interaction of treatment with prior achievement. However, when a test of general knowledge was used as criterion, a differential effect of the CEI on students with varying levels of precalculus achievement was observed to be close to significant ($\alpha = .084$). Students with high trigonometry pretest scores seemed to benefit more from the CEI than those with high algebra pretest scores. Figure 1 shows the regression planes for the CEI and Non-CEI groups and indicates the region where the expected score of the CEI group exceeds that of the Non-CEI group for different combinations of trigonometry and algebra pretest scores.

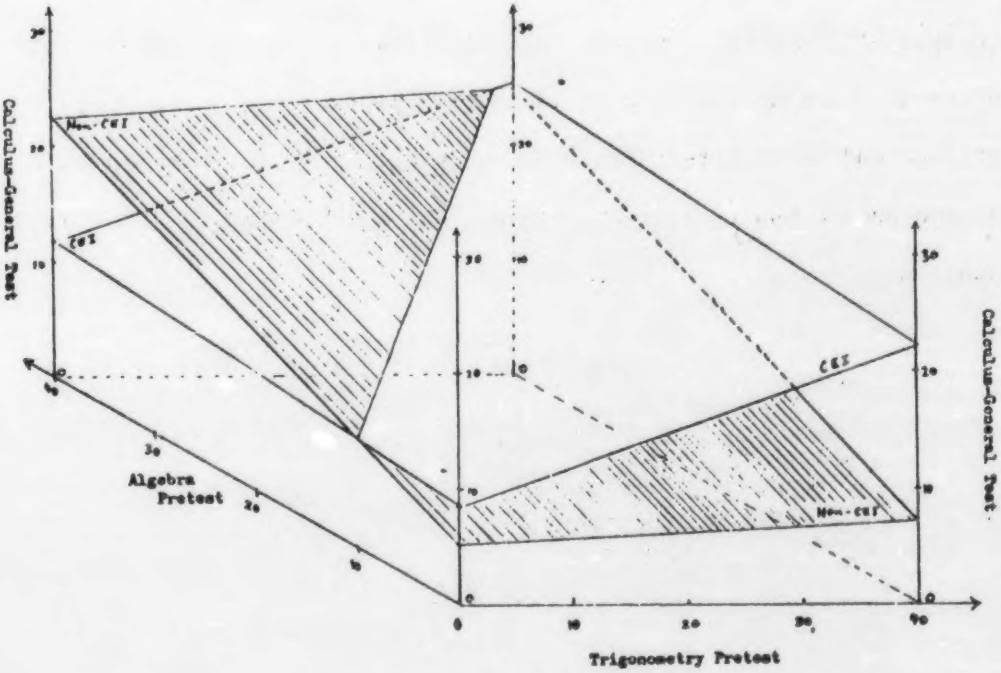

Figure 1,

Regression Planes for CEI and Non-CEI Groups

In regard to the question of the interaction of the treatment with sex classification, there was some evidence that the male students seemed to benefit more from the CEI than the female students. Even though the male students in the CEI group significantly outperformed the male students in the Non-CEI group on the Concepts Test, almost no difference in performance was evident for the female students in these two groups. These results must, however, be regarded with caution since the subsample of male students was much larger than the subsample of female studnets.

Interpretations

The fundamental conclusion of this study was that the use of this particular mode of CEI in an introductory calculus course can be an effective means of improving the student's understanding of the basic concepts of calculus. Further investigations should be conducted to determine the mode of CEI that is most propitious in various educational settings and to explore further the possible presence of an interaction between such computer augmentation and prior achievement or sex classification.

Friday, April 19, 1974
9:00 - 10:00 A.M.

Pennsylvania Room B(S)
J. Franklin Fitzgerald, Presider

RESEARCH REPORTING SECTION III

Reports: AN INVESTIGATION OF TRANSFORMATIONAL GEOMETRY INSTRUCTION IN THE MIDDLE SCHOOL

HAROLD J. WILLIFORD, Georgia State University, Atlanta, Georgia

AN INVESTIGATION OF THE RELATIONSHIP BETWEEN STUDENT ACHIEVEMENT AND SELECTED FACTORS OF THE CLASSROOM DISCOURSE

EARL W. SWANK, Coastal Plains Cooperative Educational Services Agency, Valdosta, Georgia

THE EFFECT OF ADVANCE ORGANIZERS UPON THE LEARNING OF SKILLS IN ELEMENTARY ALGEBRA

H. L. THOMAS, State University College, Oneonta, New York

C. J. LOVETT, Newark State College, Union, New Jersey

T. MCMULLIN, Vestal Central Schools, Vestal, New York

A FORMATIVE EVALUATION OF DIFFERENT APPROACHES TO TEACHING ADDITION AND SUBTRACTION TO SECOND GRADERS

WAI-CHING HO, Educational Research Council of America, Cleveland, Ohio

WAYNE H. MARTIN, National Assessment of Educational Progress, Denver, Colorado

SARITA SCHROCK, Educational Research Council of America, Cleveland, Ohio

AN INVESTIGATION OF TRANSFORMATIONAL GEOMETRY INSTRUCTION IN THE MIDDLE SCHOOL

Harold J. Williford
Georgia State University

Purpose

The purpose of this study was to evaluate a curriculum project concerning transformational geometry instruction at the middle school level. The following questions were examined: (1) To what degree do middle school students from urban and suburban environments acquire transformational geometry concepts; and (2) What are the transfer effects of transformational geometry instruction on students' spatial abilities.

Rationale

Many recommendations have been made concerning the study of transformational geometry in the public schools. Most reports indicate that the topic, is indeed, appropriate for a wide variety of age and ability levels, however little research has dealt with instructional questions involving subjects from both urban, or inner-city, and suburban settings. A number of studies have dealt with the general prediction of success in the study of geometry. One conclusion is that spatial ability, the ability to hold an object in mind and mentally manipulate it, is possibly related to geometry achievement (Sowder, 1973). If this observation is true then perhaps transformational geometry instruction can improve performance on measures of spatial ability. Previous research with second and third grade children did not confirm such an implication (Williford, 1972).

Research Design and Procedure

A unit of transformational geometry instructional materials, related achievement test items, and a space test were developed. The space test consisted of four 10 item subtests each dealing with a slightly different aspect of spatial ability. Some ideas for the geometry instructional and test materials were adapted from the UICSM series Motion Geometry (Harper & Row, 1969) and the SMP materials (Cambridge University Press, 1969). Other ideas were created by the author, a graduate assistant, and a middle school teacher. The final instructional unit included a collection of well detailed lesson plans designed for approximately 15 to 20 half-hour to 50 minute instructional sessions with middle school subjects, a collection of 32 worksheet dittos for use in the lessons, and a collection of other materials including cut-outs and transparancies. Nine major instructional objectives were formulated in the unit. The achievement test included 53 test items and a ceiling raw score of 96 total points.

Four public school teachers at the middle school level (grades 5-8) from the metropolitan Atlanta area taught in their respective schools. All were trained in the use of the materials. In all, 120 students completed the entire unit. Forty of these pupils were in grades 5 or 6 from two suburban schools located in middle to upper-middle class neighborhoods. The remaining 80 subjects were from grades 7 or 8 of two urban schools drawing students from very low income populations. The mean IQ score of the suburban subjects was 122 and that of the urban subjects was 84.

Prior to the instructional treatment the achievement and space tests were administered. Over the following three to four weeks the unit was taught. This was followed by the achievement and space posttests.

Findings

(1) The results indicated that significant gains (at the .01 level) were made on both the achievement and space tests by all groups of students. As expected, the achievement gains were rather dramatic indicating that the instructional unit was an effective means of teaching transformational geometry. For the total population posttest scores for item clusters measuring each of the nine instructional objectives were significantly higher than pretest cluster scores. Although the space test gains were significant in all cases, the actual "raw score" gains on the space test were rather small.

(2) On all pretest and posttest measures the suburban students scored significantly higher than the urban students.

(3) The urban subjects showed a significantly greater gain score for the achievement test, and a greater, but non-significant, gain score for the space test.

Interpretations

The results imply that the instructional treatment was successful in teaching students aspects of transformational geometry. Also, the instruction was accompanied by significant increases in spatial ability. However, these increases were rather limited, hence further research as to the effects of transformational geometry instruction on spatial ability should be done. The finding that urban subjects gained more than suburban subjects was quite surprising in light of the fact that the mean IQ scores of the urban subjects was 84 while that of the suburban subjects was 122. On the other hand, perhaps this finding indicates inaccuracy of standardized IQ measures for some groups of students. Also, the fact that the urban subjects were older and in higher grades may tend to favor their gains in achievement. In addition, further analysis of the test results indicated that the gains made by the urban subjects were primarily on items which were directly related to instruction. The suburban students began the experiment with higher performance levels in these skill areas. Consequently, a greater percentage of the suburban students' gain was in areas which required performance on items at higher cognitive levels often requiring application of things from the unit to other situations.

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Sowder, Larry. "Discriminators for Geometry Achievement in the NLSMA Y-Population" (Paper presented at 51st annual meeting of the National Council of Teachers of Mathematics, Houston, Texas, April, 1973).

Williford, Harold. "A Study of Transformational Geometry Instruction in the Primary Grades," Journal for Research in Mathematics Education, Vol. 3, No. 4, November, 1972, (260-270).

AN INVESTIGATION OF THE RELATIONSHIP BETWEEN STUDENT ACHIEVEMENT AND SELECTED FACTORS OF THE CLASSROOM DISCOURSE

Earl W. Swank
Coastal Plains Cooperative Educational Services Agency
Valdosta, Georgia

Purpose

This research was designed to investigate the effect on student achievement when the amount of content information contained in the classroom discourse is varied; and to investigate the effect on student achievement when the amount of content related teacher-pupil verbal interaction is varied.

Rationale

The teaching of concepts occupies a large part of the mathematics teacher's time, yet there is a lack of research data to indicate what factors the classroom teacher should manipulate in order to increase his effectiveness in the teaching of concepts. One factor, the amount of content information, has not been investigated to determine if there is a relationship between student achievement and the amount of content information contained in the classroom discourse. Few research studies on teacher effectiveness have investigated a cognitive aspect, such as the amount of content information, along with an affective aspect, such as the amount of teacher-pupil verbal interaction.

Research Design and Procedure

In a pilot study the verbal behavior of six teachers was recorded and analyzed in terms of the concept moves used in teaching the same mathematical concepts. The analysis revealed that some teachers used approximately twice as many concept moves as others. If the concept moves are used as units of content information, then this finding suggests that some students were given twice as much content information as others. It would seem that such a large difference in the amount of content information could cause a significant difference in student achievement between the students receiving the high number of concept moves and the students receiving the low number of concept moves.

Another phenomenon, extent of content related teacher-pupil verbal interaction, was also identified from the analysis of the classroom discourse of this pilot study. Teachers using more concept moves also appeared to have more verbal interaction with pupils. Thus, a second factor, referred to as concept move interactions, was employed in the present study. It should be noted that only verbal interactions between the teacher and pupils which directly contributed to the development of a concept move relative to a selected concept were considered. Combining two levels of concept move interactions with two frequencies of concept moves created four distinct instructional strategies. The achievement of students across the four instructional strategies was then investigated.

Two instructional strategies were developed using the concepts of function, inverse function and constant function. The high frequency concept move strategy contained approximately 120 concept moves and the low frequency concept move strategy contained approximately 60 concept moves. The high concept move interaction strategy called for the teacher to verbally interact with the students in the establishment of concept moves, while the low concept move interaction strategy required the concept moves to be established predominantly through teacher talk.

Eighty students were selected for participation in the study on the basis of 1) a pretest to insure that the students did not already possess a knowledge of the material contained in the instructional strategies and 2) an I. Q. measure so that students on two different ability levels could be selected. The students were partitioned into eight groups of ten students such that in each group five students had I. Q. measures from 100 to 105 inclusively (Ability Level I) and five students had I. Q. measures from 111 to 123 inclusively (Ability Level II). Each of the instructional strategies was implemented twice so that twenty students participated in each instructional strategy. Each instructional session was audio recorded and analyzed daily in terms of concept moves and concept move interactions to insure that the desired instructional strategies were being implemented. The instructional strategies required five sessions of approximately twenty minutes each to complete and the investigator was the instructor.

At the completion of each instructional strategy the participating students were given a 68 item achievement test. One month later the students were given a retention test (a parallel form of the achievement test) to determine the relative effectiveness of the four instructional strategies.

The achievement and retention tests were composed of four subtests so that the relative effectiveness of the four instructional strategies could be compared at four different cognitive levels -- knowledge, comprehension, application and analysis. The scores on the achievement test, retention test, and each of the cognitive subtests were subjected to analysis of variance procedures in a 2x2x2 factorial design with individual students serving as the unit of analysis. The factorial design permitted the two factor and the three factor treatment interaction effects to be tested for significance.

Findings

A significant difference between the group means of students participating in the high and low frequency of concept move strategies was obtained on the total achievement test and the comprehension subtest. The only significant difference in group means due to the frequency of concept moves that occurred on the retention test and accompanying subtests was on the comprehension subtest. Hence the results suggested that the higher frequency of concept moves facilitated student achievement on the achievement test and also, although to a lesser degree, on some retention measures.

An inspection of a significant treatment interaction effect between frequencies of concept moves and student abilities revealed that the higher frequency of concept moves was more beneficial for the lower ability level students than for the higher ability level students.

Significant differences between the means of the two concept move interaction groups favored the high concept move interaction group on the total achievement test and the comprehension subtest. This finding supports the belief of many teachers that verbal participation by students is desirable and beneficial in terms of increasing student achievement.

Significant three factor treatment interaction effects were obtained on the knowledge and application subtests of the retention test. An examination of the 2x2x2 interaction tables revealed that in most instances a relatively large difference in achievement between the two ability level groups was present when taught by the high frequency of concept move-high level of concept move interaction or by the low frequency concept move-low concept move interaction strategies. In each case the higher ability level group achieved above the lower ability level group. Yet for the remaining two instructional strategies there was little difference in the performance of the two ability groups.

THE EFFECT OF ADVANCE ORGANIZERS UPON THE LEARNING OF SKILLS IN ELEMENTARY ALGEBRA

H. L. Thomas
State University College

C. J. Lovett
Newark State College

T. McMullin
Vestal Central Schools
Vestal, N. Y.

Purpose

The purpose of the study was to examine the effect of written advance organizers upon the learning of first year algebra skills. The skills involved techniques of factoring polynomials and related material.

Rationale

The theoretical framework for the advance organizer concept is, of course, Ausubel's theory of meaningful verbal learning (David P. Ausubel, The Psychology of Meaningful Verbal Learning). Briefly, the theory assumes that cognitive structure is hierarchically organized with general, highly inclusive concepts at the apex subsuming at lower levels concepts of greater specificity. Learning, the building of this hierarchical structure for some given meaningful material, is assumed to proceed from general, abstract concepts to more specific, less inclusive ideas. One of the roles of advance organizers is to present to the learner appropriate general concepts prior to the more specific learning material. In mathematics many operational techniques are based upon abstract concepts or principles. On the basis of Ausubel's theory it would seem reasonable to expect appropriate advance organizers to have a beneficial effect upon the learning of such techniques.

In recent years there have been an increasing number of advance organizer studies specifically related to the teaching of mathematics. The results of these studies have been too varied for definite conclusions. Most of them employed written (self-instructional) learning materials (frequently on topics outside of the usual curriculum) and a learning time of relatively short duration (usually one class period). In this study the authors employed procedures designed to obtain a closer approximation to actual classroom conditions.

Research Design and Procedure

The subjects for the study were 474 ninth grade first year algebra students in two communities near Binghamton, New York. Eight teachers and three schools were involved.

Eight specific instructional objectives were identified and six lesson outlines (one per class day) embodying these objectives were written for the teachers. In addition, one exercise per lesson was provided. For each lesson an advance organizer (AO) passage and a non-organizer (N) passage were provided. The advance organizers were written as nearly as possible in accordance with Ausubel's definition. Each passage contained highly general, abstract material directly relevant to the content of the corresponding lesson. The non-organizer passages were selected excerpts from a popular book of a "general interest" nature in mathematics. While the non-organizer could be considered motivational, it was not relevant to the task at hand. For the first lesson these passages were each about four pages in length;

for the remaining lessons, about two pages in length.

The subjects in each class were randomly assigned to one of two groups (AO or N). Each day the teacher distributed the AO and N materials to the appropriate students at the beginning of the period and allowed five minutes (ten minutes for the first lesson) reading time (out of a class time of about forty minutes). No discussion of the materials was allowed. The materials were then collected and the teacher proceeded with the lesson for the day. Other than the request that the general lesson outline be followed and the exercise sets assigned, no attempt was made to control the teacher's methods. Hence, except for the written introductory materials, subjects in both groups received (within classes) the same instruction. After six class days an immediate learning test (Test A) was administered. One week later a second form (Test B) of the test was administered.

Findings

Each teacher group was considered a replication of the experiment and a two-way analysis of variance was employed (teacher by treatment group). While the main effect for teacher was significant at the .05 level (not un-expected with this design), the main effect for the treatment group was not significant. Nor was there a significant interaction. Indeed, upon closer analysis (t-tests within each class), it was clear that any benefit the subjects derived from the advance organizer materials was no more or less than that derived from the non-organizer materials.

Interpretations

There are limitations to the study which should be mentioned. First, the topic appeared quite easy for most of the students and, hence, it may be that the advance organizers could produce only a minimal advantage. Perhaps another study involving more difficult material to be learned would yield different results. Second, since the classroom situation was not controlled beyond the lesson outlines, the teachers' own classroom methods may have been sufficient to nullify any advantage produced by the advance organizers. The use of advance organizers may need to be limited to textbooks or other written instructional materials, or to situations where the teacher-presented lesson is specified in considerable detail.

A FORMATIVE EVALUATION OF DIFFERENT APPROACHES TO TEACHING ADDITION AND SUBTRACTION TO SECOND GRADERS

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Cleveland, Ohio

Wayne H. Martin
National Assessment of Educational Progress
Denver, Colorado

Sarita Schrock
Educational Research Council of America
Cleveland, Ohio

Purpose

The present study compares five approaches to teaching addition and subtraction to the second graders. It seeks to determine how to effectively increase student competence in computation.

Research Design and Procedure

The experiment started with the instruction of addition and subtraction of sums of 11 through 18. Materials were written to provide five different approaches. Generally speaking, Approach I and Approach II had a wider coverage. While these two programs contained addition and subtraction of 3-digit numbers with carrying or borrowing, most of the other three approaches covered through 3-digit addition and subtraction without carrying or borrowing. In terms of sequence, addition and subtraction are taught separately in Approach I for sums of 11 through 18, 2-digit numbers and 3-digit numbers, and in Approach II for 2-digit numbers and 3-digit numbers. The remaining approaches expose the students to addition and subtraction simultaneously in each of these categories. With respect to strategies, Approaches III and IV emphasize the use of formal equations more than the other approaches.

Eighteen classes from 13 suburban school districts participated in the study. Approach I was used by 5 classes; Approach II by 5 classes; Approach III by 2 classes; Approach IV by 3 classes; Approach V by 3 classes.

Several kinds of instruments were used to assess the students' computational skills and ability:

1. Readiness Test: Prerequisite skills were measured to establish initial ability at the beginning of the experiment.
2. End-of-the-Year Test: The End-of-the-Year Test was designed to cover objectives in all approaches. Furthermore, maintenance of skills taught in grade 1 was also tested. The test consisted of two parts. Part I measured the addition and subtraction facts. Part II measured addition and subtraction with or without carrying and borrowing through the 3-digit numbers.
3. The Lorge-Thorndike Test: The Lorge-Thorndike Intelligence Tests were administered to all students to determine their mental ability.

Two kinds of analyses were performed to investigate effective ways to increase student's computational proficiency.

1. Comparison of the overall achievement of the various approach groups: The one-way analysis of covariance, using IQ and readiness score as the covariates, was conducted to compare the various approach groups on each of the two part scores of the End-of-the-Year Test. When the groups were found significantly

different, the Newman-Keuls tests were conducted to compare every pair of the groups.

2. Comparison of attainment of objectives by various approach groups: In order to identify the effectiveness of the various approaches with students of differing ability levels, students were classified into 3 ability levels on the basis of Lorge-Thorndike IQ scores. The average percent passing of items measuring each objective was then computed for each approach group at each ability level.

Findings

1. No significant differences were found among the various approach groups regarding their performance on the addition and subtraction facts. The mean scores of all groups were above 90% of the maximum score.
2. The different approach groups, however, were highly significant in their performance on the computation. Students using Approaches I, II and V achieved significantly higher scores than those using Approaches III and IV. There was no significant difference between the Approach II and Approach V groups; however, the Approach II group was significantly superior to the Approach I group.
3. With respect to the attainment of objectives, the Approach I group led at all ability levels.

Interpretations

It appears that several conclusions might be drawn from the findings above:

1. Use of algebraic equations does not seem to be appropriate at the second grade level.
2. Teaching addition and subtraction simultaneously does not seem to cause interference in the student's learning.
3. It appears that with appropriate instruction, second graders can be taught to compute 3-digit addition and subtraction with carrying and borrowing.

Friday, April 19, 1974
10:30 - 11:30 A.M.

Pennsylvania Room B(S)
Donovan R. Lichtenberg, Presider

RESEARCH REPORTING SECTION IV

Reports: AN INVESTIGATION OF NINE, ELEVEN, AND THIRTEEN YEAR-OLD CHILDREN'S COMPREHENSION OF EUCLIDEAN TRANSFORMATIONS

F. RICHARD KIDDER, Longwood College, Farmville, Virginia

THE EFFECT ON MATHEMATICS ACHIEVEMENT OF PROGRAMMED TUTORING AS A METHOD OF INDIVIDUALIZED, ONE-TO-ONE INSTRUCTION

NINA L. RONSHAUSEN, Indiana University, Bloomington, Indiana

RELATIONS AMONG PIAGETIAN GROUPING STRUCTURES: A TRAINING STUDY

EDWARD A. SILVER, Teachers College, Columbia University, New York

ON THE MEASUREMENT OF POLYGONAL PATHS BY YOUNG CHILDREN

TERRY BAILEY, West Georgia College, Carrolton, Georgia

AN INVESTIGATION OF NINE, ELEVEN, AND THIRTEEN YEAR-OLD CHILDREN'S COMPREHENSION OF EUCLIDEAN TRANSFORMATIONS

F. Richard Kidder
Longwood College
Farmville, Virginia

Purpose

To investigate nine, eleven, and thirteen year-old children's comprehension of Euclidean transformations at the representational level; that is, to investigate their ability to form a mental image of a planar figure, perform a mental operation (Euclidean transformation) on this representation, and then to construct the resultant in correct position.

Rationale

The substantive recommendations for the inclusion of geometry, particularly transformational geometry, in the elementary and middle school curriculum are being implemented. However, there is a scarcity of research data describing in detail the cognitive capability of elementary and middle school children on which to base these curriculum innovations. More specifically, little empirical evidence exists concerning the spatial capability of children with respect to Euclidean transformations.

In devising the study, primary consideration was given to the structure of geometry and to a comparison of this structure with what is known about the cognitive structure of the elementary and middle school child. Klein's principle, with its emphasis on transformations, invariants under transformations, and groups of transformations concisely revealed the structure of geometry. The child's cognitive development was examined via the developmental theory of Jean Piaget. According to Piaget, the evolution of spatial relations proceeds at two different levels -- at the perceptual level and at the level of thought or imagination (representational space). Piaget posits that the child's spatial representations are built up through the organization of mental "actions" performed on objects in space, that gradually these actions coalesce to form systems of actions, and that by the age of 9-11 the framework appropriate to comprehensive Euclidean and projective systems are developed. Hence, it was hypothesized that: (1) thirteen year-old children can perform Euclidean transformations, compositions of transformations, and inverse transformations at the representational level; (2) the ability to perform transformations, compositions of transformations, and inverse transformations is age related; and (3) the ability to perform transformations, compositions of transformations, and inverse transformations at the representational level is related to the ability to perform these operations at the perceptual level.

Research Design and Procedure

Ninety subjects, thirty each from grades four, six, and eight participated in the investigation. Participants were first administered a paper and pencil perceptual-recognition test in groups by grades. A training session on sliding, flipping, and turning a rigid copy of an original figure (operational definition of the three basic motions) was held individually with each subject. Immediately following the training session, using Piagetian-like tasks, those subjects who evidenced understanding of the "operational definition" of at least one motion were then tested on the three individual motions, six of the nine possible compositions, and the three inverse motions at the representational level (referred to as transformational tasks). Approximately three months after individual testing all subjects were administered the Otis-Lennon Mental Ability Test, Form J.

The transformational tasks consisted of three subtests -- individual motions, compositions, and inverse motions. The scores on the perceptual-recognition test and the three subtests of the transformational tasks were subjected to an analysis of variance (an Age-group X Motion repeated measurements design) to determine if age and/or motion were significant factors in the subject's ability to perform transformations, compositions, and inverse transformations. The percentage of subjects satisfactorily performing the transformational tasks was computed and used to assess an age-group's ability to perform transformations, compositions, and inverse transformations at the representational level. Pearson's product moment coefficients were computed to examine relationships between perceptual spatial ability and representational spatial ability with respect to Euclidean transformations. Subject's errors on each motion, composition, and inverse motion were tabulated to determine the kind and amount of errors committed by individuals and the type of errors occurring on each motion.

Findings

The ANOVA revealed that motion was a significant factor ($p < .01$) in both spatial levels of testing, with translations being easier to perform than either reflections or rotations. Surprisingly, only in the perceptual-recognition test and the individual motions subtest of the transformational tasks was age found to be significant. The perceptual-recognition test scores (perceptual ability) were not highly related with performance on the transformational tasks (representational ability). However, the data indicated that a marked relatedness existed between the scores on the Otis-Lennon Mental Ability Test and the ability to perform transformations at the representational level. It appears that in performing transformations at the representational level, general mental ability is at least as important a factor as is age.

It was found that the subjects had difficulty in performing Euclidean transformations in both modes of testing. In particular, over 50% of subjects in each age group failed to perform transformations, compositions of transformations, and inverse transformations at the representational level. No error pattern could be established. However, 60% or more of all errors committed on the transformational tasks were "failures to conserve length" (did not construct a congruent image of the original figure). Strikingly, only six percent of the subjects attempting the transformational tasks were able to perform all twelve individual motions with no conservation of length errors.

Piaget and others have found that children conserve length by six to eight years of age. It was conjectured that the failure to conserve length by the present subjects was due to the difference in the tasks used to measure "classical" conservation of length and the tasks of the present study. That is, the simplicity of the "classical" tasks as compared with the complexity of performing Euclidean transformations. The large number of "failure to conserve length" errors and the almost complete lack of error pattern strongly suggests that the ability to perform Euclidean transformations at the representational level derives from formal-operational thought (in a Piagetian sense) and that the thirteen year-old subjects of the present study were not in the formal-operational stage. One general property of formal-operational thought is the ability to separate out factors according to combinations not given by direct observation and the necessary tools for holding one factor constant in order to determine the causal action of another. It appears that just such a capability is needed to perform Euclidean transformations at the representational level.

THE EFFECT ON MATHEMATICS ACHIEVEMENT OF PROGRAMED TUTORING AS A METHOD OF INDIVIDUALIZED, ONE-TO-ONE INSTRUCTION

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Indiana University

Purpose

The purpose of the study was to investigate the effectiveness of programed tutoring as a method of providing individualized, one-to-one mathematics instruction in kindergarten and first grade.

Rationale

Programed tutoring was developed originally as a method for teaching beginning reading. It has proved to be an effective instructional method, particularly for children who appear to have the greatest difficulty learning in the regular classroom situation. It is important to note that results have indicated that the effectiveness of the method is due to the materials and strategies of programed tutoring and not just to one-to-one instruction or additional instructional time. Programed tutoring combines some principles of programed instruction with some currently-accepted classroom practices.

Tutoring is controlled by two sets of programs. The operational programs provide tutoring strategies. The content programs specify what is to be learned and the sequence in which the tasks are to be presented. The operational programs describe correct and incorrect responses; the content programs include specific correct responses. Therefore, persons with limited knowledge can perform successfully as tutors.

Research Design and Procedure

The mathematics content provided included eleven major topics -- sets and subsets, union and intersection of sets, one-to-one matching and set relations, cardinal numbers, addition of cardinal numbers, subtraction of cardinal numbers, numeration and place value, equality and number sentences, basic properties of addition, telling time on the hour and half-hour, and values of sets of coins. No attempt was made to adopt the development or sequencing of topics of any commonly-used basal series, but the scope and sequence of the content programs is similar to many others.

Six assumptions based on earlier studies of programed tutoring were made in order to implement this study: (a) programed tutoring should be used as a supplement to the regular classroom instruction in mathematics; (b) one tutoring session daily is about as effective as two sessions daily and due to the cost only one session is given; (c) the optimum length of the tutoring session is 15 minutes; (d) replacement of a tutor during the school year has no effect on achievement (there is rarely more than one replacement of a tutor during the year and usually no replacements at all); (e) the effectiveness of programed tutoring is due to the materials and strategies employed, not to one-to-one instruction or additional instructional time; (f) every success should be rewarded (100% reinforcement).

The study was conducted in two large city school systems -- the Indianapolis (Indiana) Public Schools and the Sacramento (California) City Unified School District. In each school system, 14 elementary schools participated. Stratified random samples of first-graders were formed using total scores on the Metropolitan Readiness Tests. Stratified random samples of kindergarten classes in two Sacramento schools were formed by the same procedure. Comparable control groups for each of the three experimental groups (Indianapolis first grade, Sacramento first grade, Sacramento kindergarten) were formed by the same procedure at the time the experimental groups were formed.

In Indianapolis, 206 first-graders were selected for the experimental group and 98 were selected for the control groups. In Sacramento, 155 first-graders and 33 kindergarten children were selected for the experimental groups; 86 first-graders and 17 kindergarten children were selected for the control groups. There were no significant differences in mean pretest scores for any pair of experimental and control groups when tutoring began. The comparisons of pretest scores were repeated at the end of the school year, using only scores of subjects who completed all posttesting; again, there were no significant differences in mean pretest scores for any of the three pairs of experimental groups.

Children in the experimental groups received tutoring in mathematics for their grade level for 15 minutes each school day as a supplement to their regular classroom instruction. Children in the Indianapolis tutoring project were tutored seven months; children in the Sacramento project were tutored four months. Children in the control groups received only the regular classroom instruction in mathematics. At the end of the school year, first grade children in the experimental and control groups were given the mathematics subtests of the Metropolitan Achievement Tests, Primary I. The first grade children in Sacramento were also given the Criterion Mathematics Test, Levels I-IV, a criterion-referenced measure developed within the school system. Kindergarten children were given the Metropolitan Achievement Tests, Primer and the Criterion Mathematics Test, Level I.

Because treatment-readiness level interactions were reported in some earlier studies of programed tutoring, scores obtained on the posttest measures given to children in Indianapolis were analyzed in a two-factor ANOVA design. The factors were treatment (T_1 - tutored, T_2 - untutored control) and readiness level (P_1 - high, P_2 - middle, P_3 - low). The readiness levels were defined systematically in terms of total scores on the pretest. Scores obtained on the posttest measures given to children at both grade levels in Sacramento were analyzed by a two-tailed *t*-test.

Findings

No treatment-readiness level interactions were found when the posttest scores obtained in Indianapolis were analyzed. On each subtest, the mean score obtained by the tutored group exceeded the mean score obtained by the untutored control group. These differences reached the .025 level of significance for the computational skills subtest and the combined computational skills and concepts subtests. The difference reached the .10 level of significance for the concepts test.

When the posttest scores obtained by the first-graders in Sacramento were analyzed, all the differences in mean scores favored the tutored group over the untutored control group. About one-half of the differences could be considered statistically significant. The difference in mean scores on the computational skills subtest of the Metropolitan Achievement Tests reached the .10 level of significance. The difference in mean scores on the combined mathematics subtests of the Metropolitan, Level IV of the Criterion Mathematics Test and the combined subtests of the Criterion Mathematics Test each reached the .20 level of significance. No other differences were considered statistically significant.

While the difference in mean scores obtained on the posttest measure by the kindergarten groups in Sacramento favored the tutored group in each case, only one difference can be considered statistically significant. The difference in mean scores on the computational skills subtest of the Metropolitan Achievement Tests, Primer, reached the .10 level of significance.

Interpretations

The data indicate that programed tutoring when used as a supplement to regular classroom instruction is more effective in increasing mathematics achievement than regular classroom instruction alone. Although programed tutoring was effective as an instructional method for both mathematics concepts and computational skills, the data infer it was more effective for computational skills.

The apparent effectiveness of programed tutoring in this study is encouraging for three reasons. First, programed tutoring can be used as an intervention program to help children avoid failure in both mathematics concepts and computational skills during the first grade year. Second, the systematic mathematics instruction afforded by programed tutoring is beneficial to kindergarten children's achievement. The third reason relates to the use of programed tutoring as a research tool. Because of the control and reproducibility of conditions which are integral parts of programed tutoring, it could be very useful in research on curriculum development and teaching methods. By successfully extending the application of programed tutoring to a second content area, a big step in its refinement as a research tool has been achieved.

RELATIONS AMONG PIAGETIAN GROUPING STRUCTURES:
A TRAINING STUDY

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Teachers College, Columbia University

Purpose and Rationale

Piaget's theory of concrete operational behavior describes certain logico-mathematical structures called groupings which correspond to the organization of children's cognitive and behavioral actions.

Research information related to the grouping structures has been obtained largely through the use of two different general strategies. The first strategy has been an investigation of the hierarchical nature of cognitive growth involving an examination of the stages passed through in attaining abilities. The investigations of Kofsky (1966) into classificatory development and of Wohlwill (1960) into the development of the number concept are examples of this strategy. The second general strategy has been an investigation of the integrative nature of cognitive growth involving an examination of the convergence and correspondence among distinct abilities. The investigation into convergence of conservation abilities by Beilin (1965) and the investigation by Dodwell (1962) into children's understanding of cardinal number and the logic of classes are both exemplifications of this second strategy.

It was the purpose of this study to investigate the grouping structures using a third strategy. This strategy is an investigation of the effects of training experiences involving behaviors in one grouping (primary addition of classes) on certain other behaviors (class inclusion) within that same grouping and also on certain behaviors in other groupings (number and substance conservation in the grouping of multiplication of relations; transitivity in the grouping of addition of asymmetrical relations). The behaviors and grouping under study were chosen because of the existence of a large body of research related to them which might aid in the analysis of the results, as well as their importance within Piaget's theory and their importance in the development of early mathematical knowledge in the child.

The training technique used was a modification of the highly verbal, multiple classification and labeling method of Sigel, Roeper, & Hooper (1966), who reported success in increasing the conservation ability of trained Ss.

Since, for Piaget, all the behaviors classified by the same grouping share a common logical structure, it would be expected that the behaviors within the same grouping as the trained behaviors would be strengthened. Evidence is available, however, to support two conflicting hypotheses regarding the effects of training on behaviors in other groupings. Since the groupings each have their own distinct logical existence, it might be expected that training within one grouping would have little effect on behaviors in other groupings. On the other hand, there is evidence to support the expectation that training in one grouping may affect behaviors in other groups (see Shantz, 1967 and Flavell, 1963, p. 198). These conflicting hypotheses were investigated in this study.

Research Design and Procedure

The experiment consisted of three phases: pretesting, training, and posttesting. The Ss ranged in age from 6 years, 3 months to 7 years, 6 months. In pretesting, the 55 Ss were given standard number and substance conservation, class inclusion, and transitivity tests, consisting of three items for each. For each item, correct judgments and adequate reasons each received 1 point, and incorrect judgments and inadequate reasons received no points. No reasons were asked for on the transitivity test. After pretesting, the Ss were matched for age and total number of correct judgments and were assigned randomly to the Training and Control groups.

There were two training sessions, each divided into three phases. The materials used during training were 18 attribute blocks of assorted sizes, shapes, and colors. For each phase of training a criterion of 4 consecutive correct responses was used to determine movement to the next phase. Thus, the number of trials varied among subjects.

Phases 1A and 1B were designed to assess the subject's knowldege of the vocabulary associated with the blocks. During Phase 1C the children were asked to identify the similarities and dissimilarities between two blocks. During this phase of training, it was continually emphasized that things could be the same and different *at the same time*. Whenever S gave an incorrect response, E used a simple correction procedure and gave S the appropriate information.

Phase 2A required S to find a block which had two defining characteristics and then another with the same characteristics. During Phase 2B an attempt was made to induce cognitive conflict through a process of questioning about the fact that no two blocks could share 3 attributes. Phase 2C was identical with Phase 1C.

The members of the Control group also had two sessions with the experimenter, but there was no discussion of attributes.

There were two posttests. Posttest I was given 1-3 days following training and was identical with the Pretest. Posttest II was given 3 weeks after Posttest I and was divided into two parts given on successive days. Part A was a repetition of the pretest number and class inclusion items; Part B consisted of two new items for each. Transitivity and substance conservation were not tested in Posttest II.

Findings

The posttest data revealed that training had minimal effects on substance conservation and transitivity performance.

The performance of Training and Control subjects is summarized in Table 1. Since scores on the two posttests were almost constant, subsequent analyses compared Pretest scores with Posttest I scores only.

The effects of training on scores for class inclusion and number conservation were highly significant. For class inclusion, there were 12 positive changes and none were negative (sign test, $p < .001$); for number conservation there were 9 positive changes and no negative (sign test, $p < .01$). There were no changes for the Control group.

Training and Control Ss were matched on the basis of Pretest class inclusion scores, and trained Ss scored significantly higher on the Posttest than their Control mates (Wilcoxon Matched Pairs Test, $p < .005$). A similar analysis was done for effects on number conservation scores, and trained Ss scored significantly higher than Control Ss (Wilcoxon Matched Pairs Test, $p < .005$).

There were 8 Ss in the Training group moving from non-class-

Table 1

Performance of Training and Control Group Subjects

		Number		Class Inclusion	
		Conserver[a]	Non-conserver	Includer[a]	Non-Includer
Training Group (N=27)	Pretest	7	20	2	25
	Post I	12	15	10	17
	Post IIA	13	14	12	15
	Post IIB	12	15	12	15
Control Group (N=28)	Pretest	8	20	1	27
	Post I	8	20	1	27
	Post IIA	9	19	2	26
	Post IIB	8	20	2	26

[a] For Posttest IIB, conserver had a score of 4; for other tests, conservers had a score of 5 or 6. For class includers the same scoring was used.

includer and 5 Ss moving from non-number-conserver status; no Ss showed the reverse. The category changes, both for class inclusion (McNemar Test, $\chi^2 = 6.1$, $p < .01$) and for number conservation (McNemar Test, $\chi^2 = 3.2$, $p < .05$) were significant.

Interpretations

The procedure used during training combined elements of class addition and multiplication of relations to produce significant improvement in the class inclusion ability of trained subjects. The present study indicates that training subjects in classification and discrimination of similarities and dissimilarities and supports the hypothesis that training behaviors in a grouping tends to strengthen other behaviors within that same grouping.

The effects of training on behaviors in other groupings was more difficult to analyze. Because of the high number of correct responses to the Pretest transitivity items, training effects could not have been more than minimal. The effects of training within the conservation grouping were mixed. No change occurred in substance conservation ability; however, 5 Ss became number conservers as a result of training, and 9 Ss showed a gain in number conservation ability.

The increased success in number conservation situations for some subjects was attributed to their increased class inclusion ability. The 12 Ss who showed an increase in class inclusion ability either gained also in number conservation ability (N=6), or were already number conservers (N=6), whereas only 2 Ss showed a gain in conservation without also gaining in inclusion ability. Both Wohlwill (1968) and Dodwell (1962) report a close relationship between these two abilities.

Both the relationship between class inclusion and number conservation and the theoretical basis for a relationship between behaviors in the transitivity grouping and behaviors in the class inclusion grouping (see Flavell, 1963, p. 198) deserve further study. It may be that there is a hierarchy, not only within each grouping, as we know exists in the conservation grouping, but also across the nine groupings.

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ON THE MEASUREMENT OF POLYGONAL PATHS BY YOUNG CHILDREN

Terry Bailey
West Georgia College

Purpose

The purposes of this study are: (1) to determine the ability of a collection of first, second and third graders to establish a length relation between two polygonal paths (both in a zigzag pattern) where the segments in each path are homogenous in length; (2) to determine the ability of these children to engage in transitive reasoning involving length relations; (3) to determine the relationship between their ability to engage in transitive reasoning involving length relations and their ability to reason using the substitution property of length relations; (4) to study the effectiveness of telling these children a needed result; (5) to determine the effect of a cognitive conflict situation in causing these children to logically reason when establishing a length relation between these paths; and (6) to study the ability of these children to conserve the length relation they established between the paths when one path is slid parallel with respect to the other path.

Rationale

Piaget strongly believes that children progress mentally by interacting with their environment. He believes that it is very ineffective to simply tell a child a needed result, concept or principle. According to Piaget, the main force in modifying a child's cognitive structure is a process called equilibration that is set off when cognitive conflict occurs. Simply confronting a child with a particular situation in order to create cognitive conflict is theoretically not enough, however, since one of the components of Piagetion theory is the assertion that a particular modification of a child's cognitive structure does not happen instantaneously and that cognitive conflict needs to happen in a wide variety of settings.

Piaget has found that children in particular age intervals usually do not conserve certain properties when these properties undergo certain transformations that leave them unaltered. Piaget says that children are able to conserve when they realize that the inverse transformation is possible. He calls this reversibility.

Suppose a length relation is to be established between two polygonal paths (both in a zigzag pattern) where the segments in each path are homogenous in length. For a child to be able to meaningfully perform measurement relative to establishing a length relation between these paths he must be able to establish a number relation, conserve number relations, establish a length relation, conserve length, conserve length relations, use the transitive property of length relations, use the substitution property of length relations and to logically multiply length of unit with number of iterations.

Four different tests were administered. In test 1, the segments in one path were longer than the segments in the other path, but were equal in number. In test 2, the segments in one path were the same length as the segments in the other path, but unequal in number. In test 3, one path had longer but fewer segments than the other path and in test 4, the segments in both paths were equal with respect to number and length. Each pair of paths protruded out on each end about the same distance. In each test, the length relation between the paths could not be determined without a measuring stick.

Forty students were randomly chosen from the top two-thirds of each of grades one, two, and three, where the top two-thirds of each grade were mainly determined by teacher judgment. In each grade, after the Ss were chosen, they were randomly assigned to one of the four tests.

Each S was given a preliminary exercise in determining length relations between pairs of dowel sticks. If E decided that S could not establish length relations with respect to these sticks, S was discarded and another S was chosen in his place. Every S was questioned by E in a one-to-one situation for about 25 minutes. The performance of each S relative to certain types of reasoning was recorded by E on a specially prepared check sheet and the dialogue between S and E was recorded by an assistant.

Findings

Some of the major findings were:

1. From the collection of Ss who did not take test 3, it was found that only 1 (5.9 percent) of the first graders, 3 (13.7 percent) of the second graders and 11 (50 percent) of the third graders conserved the length relation they established between the paths when one path was slid parallel with respect to the other path.
2. For the Ss between the ages of 7 and 8 years, the technique of giving them an appropriate measuring stick and telling them to use it seemed quite effective in helping them to engage in

transitive reasoning using length relations.

3. The ability to engage in transitive reasoning involving length relations was found to be a necessary condition to be able to logically reason using the substitution property of length relations.
4. The data indicated that the ability to simultaneously use the dimensions of length of units and number of iterations to logically establish a length relation between two polygonal paths usually appears in children beyond the age of nine years.
5. The data revealed that in establishing length relations, it is very ineffective to simply tell a child a needed result, concept or principle.
6. The data implied that a cognitive conflict situation administered once does little good in causing children who establish the correct length relation between two polygonal paths and reason using only one dimension (i.e., length of units or number of iterations) to correctly reason using both dimensions.
7. It was found that children who establish the correct length relation between two polygonal paths and reason using only the length dimension tend to conserve the length relation they established more often than the children who establish the correct length relation between two polygonal paths and reason using only the number dimension.

Friday, April 19, 1974
1:30 - 2:30 P.M.

Pennsylvania Room B(S)
Houston T. Karnes, Presider

RESEARCH REPORTING SECTION V

Reports: RECALL OF ADVANCE ORGANIZERS IN MATHEMATICS CONCEPT LEARNING

GEORGE W. BRIGHT, Northern Illinois University, DeKalb, Illinois

LONG TERM EFFECTS OF ACCELERATION ON UNDERGRADUATE CALCULUS STUDENTS IN THE CRIMEL PROGRAM AN THE OHIO STATE UNIVERSITY

LAWRENCE ALLEN COON, West Liberty State College, West Liberty, West Virginia

INFORMATION INVENTORY IN MATHEMATICS EDUCATION

SYLVIA AUTON, University of Maryland, College Park, Maryland

HARRY TUNIS, University of Maryland, College Park, Maryland

AN ATTEMPT TO CONSTRUCT A PREDICTIVE DEVICE FOR PLACING FRESHMAN STUDENTS INTO CALCULUS AT SHIPPENSBURG STATE COLLEGE

KENNETH C. WASHINGER, Shippensburg State College, Shippensburg, Pennsylvania

RECALL OF ADVANCE ORGANIZERS IN MATHEMATICS CONCEPT LEARNING

George W. Bright
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Purpose

The present study extended previous research on advance organizers and was designed to determine whether the congitive association between an advance organizer (AO) and material to be learned could be strengthened during the course of instruction. Ausubel has stated that learning might be facilitated by presenting an AO prior to instruction, with the AO being more general and more inclusive than the material to be learned. At the same time the organizer must be anchorable in the cognitive structure of the learner; for if the organizer is rotely learned and consequently unstable, then it cannot be expected to provide adequate anchorage for the material to be learned.

Rationale

The present study was designed to take advantage of the idea that an AO operates because it provides stable anchorage for concepts to be learned. The seeming ineffectiveness of AOs in previous studies might be explained in part by the fact that the relationship between the organizer and the material to be learned was not sufficiently well established. Such a lack of relationship might be caused by the physical unavailability of the organizer or by the learner's failure to recall the organizer during instruction. Increasing the likelihood of such recall might enenhance the effectiveness of the AOs.

Stated in null form, the hypotheses were as follows:

1. The level of abstractness and inclusiveness of the advance organizer does not affect learning of the concepts of integer addition among prospective elementary school teachers.
2. Repeated recall of an advance organizer during instruction does not affect learning of the concepts of integer addition among prospective elementary school teachers.

Research Design and Procedure

The learning material for this study was modified from previous research and consisted of a 13-page printed booklet on addition of integers. There were two AOs: a listing of the axioms of a mathematical field, and a definition of a mathematical system. For each, three paragraphs were written to be inserted in the booklets. These paragraphs were designed to recall the respective organizer, or parts thereof, into the cognitive field of the learner and to help relate the organizer to the material to be learned. A test of immediate learning was developed and consisted of 23 items, 12 to test computation skill and 11 to test recognition and recall of the structural properties of integer addition. These subscales were called "skill" and "theory," respectively.

The subjects were students enrolled in two sections of the first-semester mathematics content course for elementary education majors at Northern Illinois University. Seventy-seven per cent of the subjects were freshmen, and 93 percent were female.

The design was a 2 x 5 factorial design, the first factor being type of organizer and the second factor being number of instances of recall. The two organizers were used and recalled from zero to four times (hence, five levels for the second factor) with the recall passages being the paragraphs previously described. Booklets, constructed to accommodate the ten treatments, were randomly ordered and distributed sequentially to the subjects. Subjects were told that they would have one class period (50 minutes) to study the material in the booklet and to complete the test which was included in the booklet. Both hypotheses were tested for the "skill" and "theory" subscales as well as for the total score.

Findings

KR-20 reliabilities were as follows: subscale skill .690, subscale theory .665, and total .784. A 2 x 5 ANOVA was computed. None of the main effects were significant, so neither of the hypotheses could be rejected. The F ratio for the interaction term for the subscale skill was significant at the .05 level, so post hoc analyses were completed.

A plot of the group means was made. Inspection of the graphs for the AO groups revealed that five of the six graphs were roughly in the shape of a U. The occurrence of five such events independently has a probability of less than .01. Of course the observed events were not independent, so the true probability is larger by an unknown amount. To clarify this observation, the data of the eight AO groups were reanalyzed by a 2 x 4 ANOVA. F ratios for the main effect of type of AO were significant at the .025 level for both the subscale skill and the total test score. The F ratio for interaction was not significant.

The lack of significant main effects in the test of the hypotheses together with the existence of a significant main effect in the post hoc analysis suggests that the effect of the AOs among the AO groups was different than the effect between the no-AO (control) and the AO groups. This discontinuity appeared in the guise of interaction in the 2 x 5 ANOVA.

One of the most intriguing results is the seeming quadratic nature of the AO group means observed in the plot of the group means. If in fact such a description were accurate, then the linear model assumption of the 2 x 4 ANOVA ought not to be sufficient. That is, a significant F ratio for interaction should result. Since none of the F ratios for interaction were significant in the post hoc analysis, the existence of a quadratic relationship is discounted.

Interpretations

In the context of this study, it does not seem that recall of AOs enhances learning. It must be remembered, however, that the instruction was begun and concluded in less than one hour. Recall of an AO during longer-term instruction might enhance effectiveness. Too, the AOs of this study may not have been cognitively stable enough for the learners to permit meaningful recall. The enforced recall may have served to disrupt the cognitive structure, and hence to increase the difficulty of using the already unstable anchorage.

The treatments did have differential effects, and the nature of these effects should be more extensively studied.

LONG TERM EFFECTS OF ACCELERATION ON UNDERGRADUATE CALCULUS STUDENTS IN THE CRIMEL PROGRAM AT THE OHIO STATE UNIVERSITY

Lawrence Allen Coon
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Purpose

During the Autumn quarter of 1970, The Ohio State University Mathematics Department began the first phase of the CRIMEL (Curriculum Revision and Instruction in Mathematics at the Elementary Level) program. The program was designed for students unable to begin calculus directly and provided three pacing levels (accelerated, regular, and reduced) through the first two quarters of the large enrollment mathematics sequences. The purpose of this research was to investigate the long term effects of acceleration on the performance of students in the traditional calculus sequence, who covered three quarters of material (algebra and trigonometry, differentiation, and integration) in two quarters. The contribution is twofold: since little research on acceleration at the college level has been reported, it will help fill that gap in knowledge; and it will provide an empirical basis for evaluating the CRIMEL program and counseling CRIMEL students.

Research Design and Prodecure

Two types of students were directly affected by the accelerated pace: those who stayed with it and those who tried but returned to the regular pace. The effects of acceleration on the performance of these students was investigated by making the following three pairs of comparisons among four groups of students:

1. Accelerated students with stronger non-CRIMEL students who started directly in calculus, to determine if their subsequent performance was comparable.
2. Accelerated students with their CRIMEL peers (who either tried but returned to the regular pace or who could have been accelerated but chose the regular pace), to determine if their subsequent performance was harmed.
3. Students who tried but returned to the regular pace with their CRIMEL peers (who either were accelerated or could have been but chose the regular pace), to determine if trying the accelerated pace harmed subsequent performance.

In making these comparisons, two types of performance were considered:

1. Mathematical performance -- measured by the final grade for the last quarter of the sequence and the score on a test of the basic material of the last quarter written by the researcher.
2. Survival ability -- measured by the drop out rates for the entire sequence and for the calculus portion of the sequence.

Therefore, the three comparisons were made twice, to investigate the effects on mathematical performance and on survival ability.

The major procedural difficulty was making multiple comparisons. In order to minimize the chance of a type I error, Bonferroni t-statistics were used. These are based on the Bonferroni inequality:

$$1 - P(J) \geq 1 - \alpha_1 - \alpha_2 - \alpha_3 \cdots - \alpha_k$$

where

$\mathfrak{J} = \left\{ s_f \right\}$ is a family of k statements

$$\alpha_f = p\left\{ I(S_f) = 1 \right\} \quad \text{for } f = 1,2,\cdots,k$$

$$I(S_f) = \begin{cases} 1 \text{ if } S_f \text{ is true} \\ 0 \text{ if } S_f \text{ is false} \end{cases}$$

It follows that if each of k statement hold with probability $1 - \frac{\alpha}{k}$, the probability that the k statement hold simultaneously is at least $1 - \alpha$. Therefore, the problem of making k comparisons is reduced to setting the significance level, α, then testing each of the k comparisons at the $\frac{\alpha}{k}$ level. Hence, the following general procedure was used.

1. The four groups were tested for overall differences in performance at the $\alpha = 0.05$ level.
2. If significant differences were found, each of the three pairs of comparisons was tested at the 0.017 level ($\alpha/3$) to determine if the sets of students involved differed in performance.
3. Any comparisons that were significant were then tested at the 0.0083 level ($\alpha/6$) to determine if one, or both, of the criteria had a significant effect.

MANCOVA techniques were used to investigate the effects on mathematical performance. Previous studies have established that the best predictors of mathematical success at O.S.U. are ACT mathematics score, O.S.U. mathematics placement test score, and quality points (a measure of high school background). It was felt the most conservative course would be to use as many of these as possible, consistent with the requirement of homogeneity of regression. Consequently, covariates were selected as follows:

1. A MANOVA with the candidates as variates was used to order the three candidates according to the significance of the differences due to each.
2. A MANCOVA was used to make the comparison. If homogeneity of regression was not satisfied, the least significant candidate was eliminated. Repeat until equality of regression is satisfied, or all candidates are eliminated.
3. Any restrictions are applied to subsequent comparisons.

Hence the MANCOVA procedure used was derived from the general procedure by adding the covariate selection procedure.

Chi-square techniques were used to investigate the effects on survival ability. By the additive property of independent chi-square variables, the chi-square statistic for overall survival ability is the sum of the chi-squares for each drop out criterion and its degrees of freedom the sum of their degrees of freedom. Hence the chi-square procedure was derived for the general procedure by adding this modification.

Findings

The results of these analyses indicated that:

1. Though there is no direct evidence the accelerated students performed more poorly mathematically than stronger non-CRIMEL students who started directly in calculus, since their average grade was half a letter grade lower (C+ vs. B) and their average criterion test score 7% lower (58% vs. 65%) this may be the case. However, their survival ability was significantly better.

2. The mathematical performance of the accelerated students was significantly poorer than that of their CRIMEL peers due mainly to a significantly lower test score (58% vs. 72%). However, their survival ability was significantly better.
3. The overall performance, both mathematical and survival, of the students who tried but returned to the regular pace was not worse than that of their CRIMEL peers, and may have been better.

Interpretations

It was concluded that the advantages for accelerated students in CRIMEL were a lower drop out rate and an opportunity to catch up to the students who started calculus directly and the disadvantage was lower achievement. These factors must be taken into account when counseling CRIMEL students considering acceleration. In general, it appears that students who are not well enough prepared to enter calculus directly are significantly harmed by acceleration through precalculus mathematics. Hence such programs should be voluntary and the students apprised of the factors involved.

INFORMATION INVENTORY IN MATHEMATICS EDUCATION

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Purpose

Recently, guidelines for the preparation of classroom teachers of mathematics have been published by the National Council of Teachers of Mathematics. The first of the three main sections of this paper is entitled "Academic and Professional Knowledge." The competencies listed are categorized according to mathematics content and humanistic/behavioral studies. The commission charged with the preparation of the guidelines recognized that "there has been no serious attempt, however, to describe precisely how one might go about measuring these competencies. In some cases, the measurement of the attainment of the competencies is not presently possible by any objective standards." It is the purpose of this study to: 1--formulate a conceptual framework of multiple dimensions to be used in the construction of an instrument to assess a broad range of knowledge in mathematics education and 2--apply this framework in an initial effort to create a standardized information instrument. The National Teachers Examination in Mathematics is currently the instrument primarily used in such an assessment, but mathematics content and pedagogy are the only two areas tested.

Research Design and Procedure

Ten knowledge dimensions were created to account for the information presented in the following sources: textbooks on mathematics education in the secondary schools, e.g., Butler, Wren, & Banks (1970), guidelines for the preparation of classroom teachers of mathematics published by the National Council of Teachers of Mathematics (1973), and a report of a survey of content of mathematics methods courses conducted by Shafer (1970).

The ten dimensions are:

I. Curriculum Projects
II. Historical Perspectives
III. Mathematics Content
IV. Mathematics Education and Research Centers
V. Mathematicians & Mathematics Educators
VI. Organizations & Commissions
VII. Pedagogy
VIII. Research in Mathematics Education
IX. Resources
X. Testing & Evaluation

For each of the dimensions above, content was sampled from sources in the professional literature, e.g., mathematics history texts, reviews of research, clearinghouse reports on curriculum projects, recreation books, etc. Multiple choice items were constructed and administered to one group of mathematics teachers studying secondary school mathematics curriculum. After this administration an item analysis was performed and items were modified where appropriate. A second administration was made with another group of teachers in a similar course. In each of these ten areas the focal point was on the available information or knowledge base in the field. Attention to this first level of the cognitive taxonomy was believed to be an important step in investigating the preparation of mathematics teachers. Four or more items per dimension were used in this instrument.

Teachers in two sections of an education course in secondary school mathematics curriculum at the University of Maryland participated in this study; results are based on the administration of the instrument to this group (N=30).

Findings

The findings suggest that information held by the teachers in this study, on most dimensions, is not far above the 40% level, although the range was from 20-60%. Content knowledge in mathematics was at the 40% level. While the content sampled included results through the calculus, the majority of the items focused on secondary school mathematics or interpretations thereof. Thus, this percent may be viewed as indicative of secondary school mathematics knowledge.

Interpretations

There is a great need for an instrument to sample the broad area of knowledge and skills needed by the mathematics educator. This work represents the first major effort at categorizing and evaluating these concerns. With successive field use, the instrument will serve as a basis for the continued development of mathematics education inventories.

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AN ATTEMPT TO CONSTRUCT A PREDICTIVE DEVICE FOR PLACING FRESHMAN STUDENTS INTO CALCULUS AT SHIPPENSBURG STATE COLLEGE

Kenneth C. Washinger
Shippensburg State College

Purpose

The development of a test which would be used to place students into calculus or precalculus at Shippensburg State College was the primary aim of this study. Comparison of the predictive power of this test against the predictive power of a regression equation using variables which are readily available determined the relative effectiveness with which such an instrument could be used. Statistical comparisons were made between the readily available variables (SAT-M, SAT-V, Rank In Class, Average In Last Three High School Mathematics Courses, Last High School Mathematics Course, and College Major), subdivisions of the test, and regression equations using combinations of the above variables.

The hypothesis to be tested was stated as follows: The correlation coefficient relating "predictor" test outcomes with actual success in calculus will not be significantly different from the correlation coefficient relating regression equation outcomes, using predictors readily available, with actual success in calculus.

Rationale

A prestudy was completed on three hundred and thirty-three students who had taken calculus as a first course in mathematics at Shippensburg State College. Objectives were to determine the feasibility of stepwise regression procedures in prediction, the characteristics of the population, and to construct a regression equation for placing freshmen in the main study population into one of the two courses, calculus or precalculus. Stepwise regression was used on six independent variables (SAT scores, mathematical and verbal; rank in class; last mathemetics course taken in high school, senior mathematics course grade, and one dependent variable, calculus grade).

Placement of the subjects into either precalculus mathematics or calculus was determined using the regression equation developed in the prestudy. This equation is stated as follows:

$$\begin{aligned}\text{Grade in calculus} = {} & .00323\ X_1 - .00061\ X_2 + .02158\ X_3 \\ & + .04396\ X_5 + .03654\ X_6 \\ & + .50188\ X_7 - 2.95707\end{aligned}$$

where X_1 = SAT-M, X_2 = SAT-V, X_3 = Rank, X_5 = Senior Math Grade, X_6 = Senior Math Course, and X_7 = Average Math Grade.

As a result of the computations which predicted the grade of each student in calculus, students with 2.0 or better predicted grade in calculus were placed into a calculus course and students with a predicted grade of below 2.0 were placed into a precalculus course.

Research Design and Procedure

The initial population in this study was one hundred and seventy-eight freshman students entering Shippensburg State College in September 1972. Subjects had not taken any college mathematics courses previously, and were registered as mathematics, physics, biology, or elementary mathematics sequence majors. Using the regression equation developed in the prestudy, one hundred of these students were placed into calculus and seventy-eight into precalculus. After registration in courses became stable, the population of the calculus group was ninety-three. These are the subjects used in the main study.

A test specifically designed for the placement of students into calculus should be based upon the domain of prerequisite knowledge deemed necessary for success in calculus. Hence, the following procedures were used in the design and construction of the test:

1. Identification of mathematical concepts and abilities which were present or implied in the course description and/or in the content of the textbook used in calculus at Shippensburg State College.
2. Identification of mathematical abilities prerequisite to concepts and abilities found in step 1.
3. Classification of these abilities in a matrix representation as follows:

	Number Systems	Geometry	Algebra
Computation			
Comprehension			
Analysis			

4. Construction and pilot testing of test items which measure abilities in cells of the matrix.
5. Construction and pilot testing of test items which measure understanding of mathematical concepts prerequisite to calculus.
6. All questions were of multiple choice type and the test was designed so that it could be administered in fifty minutes.

This model of test construction is similar to the one used by the School of Mathematics Study Group in the National Longitudinal Study of Mathematical Abilities.

Abilities determined to be prerequisite to the content of calculus were partitioned into three broad content areas: Number Systems, Geometry, and Algebra.

Three levels of cognitive behavior were identified--computation, comprehension, and alaysis. There were eight abilities for Number Systems: three in computation, four in comprehension, and one in analysis. There were eight abilities for Geometry: three in computation, three in comprehension and two in analysis. There were fourteen abilities in algebra: three in computation, seven in comprehension, and four in analysis.

Test item scales aimed at measuring these abilities were constructed, and the initial test using these scales consisted of fifty-one items. This test was administered to one hundred and forty-seven senior high school students from five distinct high school districts. Test data were analyzed using computer techniques to determine point biserials for each item, distribution of responses, and reliability of the test. Personal interviews were held with students and teachers in these high schools to determine the content validity, appropriateness of test item vocabulary and terminology, test construction format, and so forth.

Using information and data from this analysis, test items were revised and improved. The second version consisted of forty-four items and was administered to two hundred and thirteen students who were entering freshmen in the business administration curriculum at Shippensburg State College and who had the academic high school sequence in mathematics. Data for this test were analyzed in a manner similar to the first.

The final or "predictor" test contained forty-four items. This test was administered to freshmen in all classes of precalculus and calculus in the fall of 1972 who were placed in their respective classes using the regression equation derived in the prestudy.

On the basis of test grades computed for each student in the final test, a prediction was made concerning success or failure of ninety-three students in calculus. The decision was made to predict success in calculus for any student who scored 13 or above and failure to any student who scored below 13 in the test. This decision was based upon the results of the test on the two separate sub-populations of students, those placed in precalculus and those placed in calculus. A decision regarding success or failure in calculus should be made so that the number of students which are predicted as successes and are actual failures is at a minimum, and the number of students which are predicted successes and are actual successes is at a maximum.

The mean score in the calculus section was 18 and the mean score in the precalculus section was 11. The cumulative frequency distribution of the test on the calculus section indicated that 23 percent of the students scored less than 13 on this test.

The prestudy on 333 students indicated that the percentage of students earning D or F grades was 21.3. Using this as a criteria of past performance and the information on means, standard deviations, and medians in both populations, the prediction for success in calculus using a test grade of 13 or above was determined.

Findings

The hypothesis was tested using multiple point biserial correlation procedures to determine the correlation coefficients of the "predictor" test and the regression equation B with success in calculus. The significance of difference between these two correlation coefficients was tested using the t-statistic.

The regression equation used in the hypothesis and labeled Equation B has the following independent variables: X_1 = SAT-M, X_2 = SAT-V, X_3 = Class Rank, X_4 = Average in last three mathematics courses taken in high school, X_5 = Senior mathematics course grade, X_6 = Senior Mathematics course, X_8 = Major in college. The dependent variable X_7 = grades in calculus. The seven independent variables were entered in the BMDO2R stepwise regression program and the calculus grade was entered as the dependent variable. As each variable was entered a regression equation was computed and a corresponding multiple correlation coefficient, variance, increase in variance, and F-value to enter was given. Regression Equation B computed after all variables were entered was:

$$X_7 = .00208\,X_1 + .00085\,X_2 + .02205\,X_3 - .35328\,X_4 + .59549\,X_5 + .08938\,X_6 + .0006\,X_8 - 2.22209$$

The t-statistic was computed and t = .4841. For significance at the .05 level t must be 1.9867. Therefore, the null hypothesis was accepted: the two correlation coefficients cannot be said to be significantly different. The "predictor" test and the regression equation using variables readily available were not significantly different as predictors of success or failure in college calculus.

Interpretations

Although the major objective of the study was not accomplished, the study revealed additional pertinent information concerning the predictive powers of other variables. The effectiveness of prediction of success in calculus or grades in calculus is a function of the elements and devices which are available or created for that purpose. If individual variables readily available, such as SAT-M, SAT-V, Major in college, Senior Mathematics Course, Senior Mathematics Course Grade, Rank in Class, and Average Grade in High School Mathematics, are the only predictors available, then high school rank would be the most effective predictor of both success in calculus or grades in calculus.

Combining the variables above into a regression equation, denoted by Equation B, increased the effectiveness of prediction for both success in calculus and grades in calculus. The "predictor" test designed in this study was statistically as effective a predictor of success in calculus and grades in calculus as each of the variables readily available and the regression equation B. However, using the regression equation B would yield better results in placement of students into calculus than the use of the "predictor" test results due to the reduction of error in predicting students to be successful in calculus when, in actual outcomes, the student fails calculus. This type of consideration was made throughout the study in comparing predictive techniques and their results.

The "predictor" test design enabled comparisons to be made among cognitive sections of the test (computation, comprehension, analysis) and their relationship to success in calculus and grades in calculus. This design also provided for comparisons among content sections (number systems, geometry, algebra) as related to success in calculus and grades in calculus. The cognitive division labeled comprehension was shown to have the highest correlation with both success and grades in calculus.

Improvement in the predictive power of Equation B was accomplished by addition of variables introduced by the "predictive test. The most effective predictor of both grades and success in calculus was the regression equation A described as follows:

$$X_{14} = .00155\ X_1 + .00213\ X_2 + .01568\ X_3 - .29194\ X_4 + .54898\ X_5 + .02550\ X_6 + .0342\ X_7 - .21083\ X_8 + .01357\ X_{10} + .01846\ X_{11} + .08314\ X_{12} + .58889\ X_{15} + .00283\ X_{16} - 2.5627$$

SAT-M - X_1 (Number Systems) NS - X_8
SAT-V - X_2 (Algebra) ALG - X_{10}
Rank - X_3 (Computation) COMPT - X_{11}
AVEMSM - X_4 (Comprehension) COMPH - X_{12}
SMG - X_5 (Test Prediction) PRED - X_{15}
SMC - X_6 MAJOR - X_{16}
TG - X_7 Calculus Grade - X_{14}

This improvement in predictive power was accomplished through introduction of those combinations of variables which were not highly correlated and added to the accounted-for variance.

Throughout this investigation, correlations which related various types of predictors to grades in calculus were always of greater magnitude than correlations which related the same predictors to success in calculus. Prediction procedures for success in calculus are initially dependent upon continuous variables. A dichotomous decision (pass or fail) was made for both the test and the equation, and then was compared to dichotomous decisions which were the actual outcomes in calculus. This type of procedure was necessary for prediction but decreased correlations. This difficulty does not exist when prediction was related to grades since correlations are completely determined between continuous variables. Therefore, this limitation of prediction must be recognized in this study.

Friday, April 19, 1974
4:30 - 5:30 P.M.

Gold Room (S)
Mary Ann Byrne, Presider

RESEARCH REPORTING SECTION VI

Reports: THE RELATIONSHIP OF OPEN-ENDED VS. CLOSED-ENDED MATHEMATICS LABORATORY ACTIVITIES TO THE DIVERGENT THINKING ABILITY OF PRE-SERVICE ELEMENTARY TEACHERS

JANE ANN MCLAUGHLIN, Trenton State College, Trenton, New Jersey

A STUDY OF THE INTERACTION BETWEEN PERSONOLOGICAL VARIABLES AND TWO MODES OF TEACHING A RELATION TO NINTH GRADE STUDENTS

ROBERT R. HANCOCK, Eastern Illinois University, Charleston, Illinois

INTERACTIONS BETWEEN "STRUCTURE-OF-INTELLECT" FACTORS AND INSTRUCTIONAL TREATMENT IN MODULAR ARITHMETIC - A REFINEMENT AND REPLICATION

MERLYN J. BEHR, Northern Illinois University, DeKalb, Illinois

PHILLIP M. EASTMAN, Northern Illinois University, DeKalb, Illinois

THE JOHNSON-NEYMAN TECHNIQUE: SOME CONSIDERATIONS FOR IMPROVED OR FURTHER ANALYSIS OF DATA FROM APTITUDE-TREATMENT INTERACTION INVESTIGATIONS

KENNETH W. WUNDERLICH, The University of Texas at San Antonio, San Antonio, Texas

GARY D. BORICH, The University of Texas at Austin, Austin, Texas

THE RELATIONSHIP OF OPEN-ENDED VS. CLOSED-ENDED MATHEMATICS LABORATORY ACTIVITIES TO THE DIVERGENT THINKING ABILITY OF PRE-SERVICE ELEMENTARY TEACHERS

Jane Ann McLaughlin
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Purpose

The purpose of this study was to investigate the relationship of training in and performance on open-ended vs. closed-ended mathematics laboratory activities to the divergent thinking ability of pre-service elementary teachers. The study was conducted during the Fall Semester, 1971, at Trenton State College, Trenton, New Jersey. The eighty-two students were enrolled in an elementary mathematics methods course.

Research Design and Procedure

The Torrance Test of Creative Thinking, Verbal Form A, was administered the first meeting of the semester. On the basis of these scores, the subjects were divided into three groups of divergent thinking ability. One-half of each group was then assigned randomly to one of two treatment groups. One treatment group participated in open-ended mathematics laboratory activities during the semester; the other treatment group engaged in closed-ended mathematics laboratory activities. At the end of the semester an attitude inventory, a performance test, and a preference inventory were administered to the subjects.

The hypotheses tested were:

H 1(a) The higher a person's divergent thinking ability, the better he does on open-ended mathematics laboratory activities.

H 1(b) The lower a person's divergent thinking ability, the better he does on closed-ended mathematics laboratory activities.

H 2(a) Persons high in divergent thinking ability do better on open-ended mathematics laboratory activities if they have been taught by means of open-ended activities than if they have been taught by means of closed-ended activities.

H 2(b) Persons low in divergent thinking ability do better on closed-ended mathematics laboratory activities if they have been taught by means of closed-ended activities than if they have been taught by means of open-ended activities.

H 3(a) Persons high in divergent thinking ability have greater preference for open-ended mathematics laboratory activities if they have been taught by means of open-ended activities than if they have been taught by means of closed-ended activities.

H 3(b) Persons low in divergent thinking ability have greater preference for closed-ended mathematics laboratory activities if they have been taught by means of closed-ended activities than if they have been taught by means of open-ended activities.

Other questions that were investigated included:

1. Is there a relationship between divergent thinking ability and attitude toward mathematics?
2. Is there a relationship between divergent thinking ability and preference for open-ended or closed-ended mathematics laboratory activities?

Findings

The results indicated that divergent thinking ability and performance on both open-ended and closed-ended mathematics activities were independent. The degree of performance on either the open-ended or closed-ended mathematics activities was not related to divergent thinking ability and the manner in which the subjects had been trained.

The subjects high in divergent thinking ability who had been trained in open-ended mathematics activities showed a greater preference for open-ended mathematics activities than the subjects high in divergent thinking ability who had been trained in closed-ended activities. Although the difference was not significant at .05 level, the subjects low in divergent thinking ability who had been trained in closed-ended mathematics activities tended to have a greater preference for closed-ended mathematics activities than the subjects low in divergent thinking ability who had been trained in open-ended activities.

No association was observed between divergent thinking ability and attitude toward mathematics. The data also indicated that divergent thinking ability and preference for open-ended or closed-ended mathematics activities had no relationship to each other.

A STUDY OF THE INTERACTION BETWEEN PERSONOLOGICAL VARIABLES AND TWO MODES OF TEACHING A RELATION TO NINTH GRADE STUDENTS

Robert R. Hancock
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Purpose

The study described in this abstract was designed to investigate the interaction between the personological variables of sex-difference and certain mental factors (as defined by Guilford in his Structure-of-Intellect model) with two modes of teaching selected aspects of a contrived mathematical relation to ninth-grade students. It is felt that research of this type is in the mainstream of current educational efforts to individualize the instructional processes in the sense that (1) it seeks to identify personological variables which may be significantly related to differing cognitive preferences and (2) it attempts to shed light on the question of whether or not instructional materials can be prepared which will interact with different profiles of individual aptitudes.

Rationale

Contemporary educational thinking emphasizes the desirability of individualizing instruction in order to maximize 'payoff' in terms of achievement, retention, and transfer of training. Paradoxically, however, educational research--until recent years--has tended to deal largely with questions related to the significance of the differences between group mean scores; aptitude-treatment-interaction (ATI) research, on the other hand, seeks to provide a basis for employing differentiated treatments with a view to exploiting cognitive preferences displayed by different individuals for differing content and/or mode of instruction. The ATI paradigm first proposed by Cronbach (1957) is well known and will not be discussed here.

Research Design and Procedure

Subjects in this study were 119 ninth-grade students from seven different sections of mathematics in a local junior high school and were enrolled in either general mathematics, regular algebra, or accelerated algebra. Ss were given a battery of nine tests designed to measure cognitive ability in dealing with figural, semantic, and symbolic content. Reliabilities in this battery, as reported by Guilford et al., ranged from 0.59 to 0.78 and the scores of these tests were treated as the independent variables of the study.

Ss were randomly assigned, by sex, to one of two treatment groups. Group mean differences were tested and found not to be significantly different for any of the independent variables. Also, F-tests of the ratio between group variances revealed no essential differences in the variances of scores on the battery of tests. Furthermore, it was determined that the treatment groups were also homogeneous with respect to both means and variances on I. Q. scores and on Standard Achievement Scores in Mathematics as measured by the Iowa Test of Mental Ability. Hence, it was concluded that the experimental groups were well matched for purposes of the regression analysis which was to be carried out.

For three forty-three (43) minute class periods Ss studied one of two programmed units prepared for this investigation. The content of these programs dealt with a contrived mathematical relation that had all of the characteristics of a linear order relation; however, this fact was carefully disguised by the definitions employed and by the notation adopted.

One program presented the material employing a verbal mode of explication, the other was based upon a figural mode. Both programs had been examined by a group of colleagues with expertise in mathematics education and judged to be of parallel content and to conform to the specified instructional mode for each of the respective programs. Results of a pilot study had shown that both programs were effective teaching devices.

Dependent variables in this study were obtained from (1) scores on a learning test administered on the day following completion of study of the instructional materials and (2) scores on a test of retention given four (4) weeks later. Both sets of criterion measures consisted of a total score and two (2) subtest scores which measured achievement at two different levels of cognition. These levels were adopted from Bloom's taxonomy of educational objectives in the cognitive domain. The criterion tests had also been examined by a panel of colleagues in mathematics education and had been judged to be of high content validity and equitable to both treatment groups. It was also determined that the various experimental groups were homogeneous with respect to the variances of dependent variable scores, and thus satisfied the assumptions underlying the subsequent regression analysis.

The search for aptitude-treatment-interactions was carried out by techniques of regression analysis applied to the pairs of linear regression equations which had been determined for pairs of experimental groups.

Findings

It was found that, generally, treatment main effects were not significant. However, a notable exception was found in superior female performance on five of the six dependent variables.

The appearance of several disordinal interactions was an encouraging result. There were 13 instances of disordinal interactions on tests of retention but only 4 on tests of immediate learning. This belated appearance of significant interactions may indicate that, over time, those students who studied an instructional program consistent with their cognitive preferences were better able to assimilate the material into their own cognitive structure than were those students who did not.

Of the interactions which involved a Structure-of-Intellect factor, 11 were related to semantic factors, 4 involved symbolic factors, and only 1 was related to a figural factor. Thus, within the framework of the SI model, it appears that the Semantic subcategory along the Content dimension may hold promise for the selection of mental factors related to ATI.

Disordinal interactions that were related to sex differences reveal that 8 of 9 such interactions involved male *Ss*. It is not immediately clear why this is so. However, if one recalls the general superiority demonstrated by female *Ss*, a tenable hypothesis might be that boys of this age are more apt to apply themselves to a learning task if the instructional material seems consistent with their cognitive preferences. Such a conjecture is highly speculative, but it seems clear that the variable of sex-difference should continue to be incorporated into ATI research.

INTERACTIONS BETWEEN "STRUCTURE-OF-INTELLECT" FACTORS AND INSTRUCTIONAL TREATMENT IN MODULAR ARITHMETIC - A REFINEMENT AND REPLICATION

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The purpose of this study is to refine and replicate some of the findings of Behr (1970). Behr's original study was conducted in an attempt to show the existence of aptitude x treatment interactions between "Structure-of-Intellect" factors and instructional treatment in modular arithmetic. Behr hypothesized that scores on a verbal measure would predict success on a verbal treatment of modulus seven arithmetic and that scores on a spatial visualization measure would predict success on a figural treatment of modulus seven arithmetic. Behr's hypotheses were partially confirmed using simple linear regression analysis and a decision rule for determining when aptitude treatment interactions existed.

The refinement of the original study has centered around two areas. The first refinement is in the area of the aptitude measures selected and the treatment itself. In the original study, fourteen mental factors were selected for investigation. In the replication study three of the original fourteen were selected and one more factor was added making a total of four mental factors investigated. These factors were selected on the basis of the results of the original study and because of their use in other (e.g. Eastman, 1972) similar studies. The instructional materials were then rewritten in an attempt to have them conform more exactly to the aptitudes being measured. This refinement allows more generalizing and integrating the results of this study with recent studies in the same area. The second major refinement in the study is in the area of the data analysis. In Behr's original study the analysis was conducted using simple linear regression analysis. In the study proposed here, the analysis will be conducted using multiple linear regression and the statistical test for homogeneity of regression. It was felt by both authors that this method of analysis was more appropriate for the type of study being considered.

The experiment is to be carried out in three parts. The first part will consist of administering aptitude measures to approximately 200 pre-service elementary teachers. Part II consists of studying the programmed instruction learning materials for one class period. Part III consists of a retention and transfer test given one week following the instruction. All Ss will be randomly assigned to one of two treatment groups, a verbal symbolic group or a figural symbolic group.

This study is currently in progress. Exact design and procedure along with findings and interpretations will be presented at the research reporting section.

THE JOHNSON-NEYMAN TECHNIQUE: SOME CONSIDERATIONS FOR IMPROVED OR FURTHER ANALYSIS OF DATA FROM APTITUDE TREATMENT INTERACTION INVESTIGATIONS

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The University of Texas at San Antonio

Gary D. Borich
The University of Texas at Austin

Aptitude-treatment interaction (ATI) research continues to be especially popular among the research community in mathematics education. Carry (1969), Becker (1970), Behr (1970), Hernandez (1973), Eastman and Carry (1973), and Webb and Carry (1973) are examples of research and theoretical models for conducting ATI research in mathematics-education. The analysis of data obtained in such research has been and continues to undergo changes leading to more sophisticated and more decision-oriented interpretations. The Johnson-Neyman (J-N) technique (Johnson and Neyman, 1936), although not new, has become increasingly more appropriate as high-speed computing facilities have become commonplace.

The J-N technique is a means of determining regions of significance, somewhat like computing a confidence interval for a mean. The results of this technique allow one to determine those aptitute scores for which the treatments produce mean effects which differ significantly. Borich (1971), Borich and Wunderlich (1973), and Wunderlich and Borich (1973) have designed computer routines which incorporate a complete analysis of data for the case of one predictor-one criterion, assuming a linear relationship, two predictors-one criterion, assuming a linear relationship, and one predictor-one criterion, assuming a quadratic relationship, respectively. The analysis includes testing for homogeniety of variance for the criterion and for homogeniety of regression lines or planes, computing the J-N regions of significance and plotting a scatterplot of the data, regression lines, and regions of significance.

Examples of data analyzed using these routines with the J-N technique included will be displayed and discussed. Interpretation of results obtained through use of these techniques will be undertaken. Examples will be either from data collected previously in mathematics-education research or from data discussed during the 1974 NCTM Annual Meeting. A review of the assumptions underlying use of the test for homogeniety of regression and the J-N technique will be undertaken. The advantages of the use of this approach will be discussed and the increased use of the technique in ATI research encouraged.

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Saturday, April 20, 1974
10:30 - 11:30 A.M.

Pennsylvania Room B(S)
Leslie P. Steffe, Presider

RESEARCH REPORTING SECTION VII

Reports: TEACHER PREPARED LEARNING PACKAGES: AID TO STUDENT? OR TEACHER?

HAROLD L. SCHOEN, Virginia Polytechnic Institute and State University, Blacksburg, Virginia

ROBERT M. TODD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia

TOWARD A THEORY OF SEQUENCING: AN EXPLORATION OF THE EFFECTS OF INSTRUCTIONAL SEQUENCES EMPLOYING ENACTIVE AND ICONIC EMBODIMENTS ON THE ATTAINMENT AND GENERALIZATION OF CONCEPTS EMBODIED SYMBOLICALLY

EDWARD C. BEARDSLEE, Seattle Pacific College, Seattle, Washington

GERALD E. GAU, Elizabethtown College, Elizabethtown, Pennsylvania

RALPH T. HEIMER, The Pennsylvania State University, State University, Pennsylvania

AN ANALYSIS OF CONTENT STRUCTURE AND COGNITIVE STRUCTURE IN THE CONTEXT OF A PROBABILITY UNIT

WILLIAM E. GEESLIN, University of New Hampshire, Durham, New Hampshire

DEVELOPMENTAL ASPECTS OF MATHEMATICAL PROBLEM SOLVING

FRANK K. LESTER, Indiana University, Bloomington, Indiana

TEACHER PREPARED LEARNING PACKAGES:
AID TO STUDENT? OR TEACHER?

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Purpose

Two main questions were considered: (a) Does the detailed preparation of a concept centered individualized learning package (I.L.P.) by a teacher improve the teacher's ability to teach that concept using either a teacher centered (T.C.) approach or a learning package approach (as measured by student achievement)? (b) Is there a difference in student achievement scores on concepts taught by I.L.P as compared to a T.C. approach?

The objectives of the study were to develop materials, teacher capabilities, teaching methods, and a research design which would answer these questions. Some relationships among teacher, student and method variables were also of interest.

Rationale

Two areas of research form a background for this study--teacher education and instructional theory. In the first area, professional teacher preparation has traditionally been rather haphazard and lacking in a theoretical base (Conant, 1963). Some recent research, however, indicates that specific analysis of content to be taught is a valuable experience for pre-service and in-service teachers. In particular, it was found that students whose teachers were trained to write behavioral objectives achieved "significantly higher scores on subjects of computation and concepts" than those students whose teachers had no such training (Piatt, 1969). This study carries the specific training of the teacher a bit further, requiring that they write not only behavioral objectives for a concept, but also a detailed learning package.

The question of individualized instruction methods versus teacher centered methods is an old one. Some studies of this general type though not specifically relevant to this study, were conducted as early as the 1930's. Of twenty-three studies located, six indicated that the individualized approach was superior, six indicated that the traditional approach was superior, eleven reported no significant difference, and all twenty-three recommended that further research was needed. Of course, the type of "individualized" approach was not the same in all the studies. In fact, it was obvious that a specific definition of the individualized, as well as teacher-centered, approach which is consistent within treatment groups was needed. This was a major goal of the researchers in this study.

Research Design and Procedure

Six junior and senior high school mathematics teachers were identified and paired. The pairings were determined by the comparability of classes. Each teacher in each pair had two classes of mathematics at a certain level and his "mate" had two classes at the same level. In particular, each teacher in pair one taught two ninth grade General Mathematics classes, pair two taught two Algebra I classes and the third pair taught eighth grade General Math classes. Thus, six teachers and twelve classes were involved in the experiment.

Four of the six teachers were enrolled in a graduate seminar in writing individualized learning packages for secondary mathematics. The other two attended several sessions with the researchers to learn the techniques as described in Kapfer and Ovard (1971). Each pair of teachers identified two concepts (involving about a week of instruction per concept, the second following logically from the first) which they had not taught their classes but were due to be taught before the end of the school year. One teacher in the pair wrote a learning package for the first concept and the other teacher wrote one for the second concept (but not for the first).

A teacher-centered approach was specifically defined and a training session for the teachers familiarized them with this approach. All the details would be too lengthy to include here but the method included a specified amount of time spent each day on lecture and/or discussion of old material and new material with time for problem solving. In addition, homework assignments were specified.

By a flip of a coin, one of the two classes of each teacher was taught I.L.P. on the first concept and T.C. on the second. The order of treatments was reversed for that teacher's other class. Thus, each class was taught one unit by I.L.P. and the other unit by T.C. The pretests and posttests which were prepared by the teacher for each I.L.P. were administered, not only to the class being taught by the I.L.P., but also to the class being taught T.C. by the same teacher. Thus, if teacher 1 (teaching classes C_{11} and C_{12}) was paired with teacher 2 (teaching classes C_{21} and C_{22}), the treatment can be described by Figure 1.

It should be emphasized that every attempt was made by a teacher

	Unit 1	Unit 2
I.L.P.	C_{11} C_{21}	C_{12} C_{22}
T.C.	C_{12} C_{22}	C_{11} C_{21}

Figure 1

teaching a unit T.C. to meet the objectives of the I.L.P. for that unit whether or not he had prepared it.

Comprehensive data were collected prior to the study concerning the characteristics of the teachers and students involved in the experiment. The schools involved were in the Northern Virginia suburbs of Washington, D.C. The data used to test the hypotheses were mean change scores of classes from pretest to posttest on each unit. The tests were all teacher prepared as part of the I.L.P.'s.

Findings

Since the classes had been meeting for more than a semester prior to this study, the classes, not students, were used as statistical units. Standardized mean change scores (Z-scores of posttest minus pretest) were computed for each class on each unit. The main effects A (I.L.P. vs. T.C.) and B (preparer of I.L.P. vs non-preparer) and interaction, A x B, were analyzed using a 2 x 2 factorial design. The results of the ANOVA are summarized in Table 1.

Table 1

Source of Variation	Mean Square	F	d.f.
A	.2688	2.63	1
B	.6348	6.22*	1
A x B	.1020	1.32	1
Error	.0772		20

The B main effect is significant at the .05 level. This difference favored the preparer of the I.L.P. over the other teacher regardless of the teaching method used. A priori teacher, I.L.P., and class difference were spread among the cells; therefore, this difference must be due to the difference in teacher preparation.

Interpretations

The findings imply that detailed analysis of subject matter (at least mathematics) is helpful for teachers. Teacher educators should be particularly interested in these findings.

Concerning the I.L.P. versus T.C. approach little can be said. However, this study should be followed up with further research. Perhaps the I.L.P. approach cannot claim superior student achievement but its value must be measured in other ways. Or perhaps the contribution of I.L.P. is its emphasis on detailed content analysis not its implication in the classroom. If the latter is the case, then purchasing commercially prepared individualized systems will not be helpful to schools.

Rather the teachers should become actively involved in the construction of I.L.P.'s.

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TOWARD A THEORY OF SEQUENCING: AN EXPLORATION OF THE EFFECTS OF INSTRUCTIONAL SEQUENCES EMPLOYING ENACTIVE AND ICONIC EMBODIMENTS ON THE ATTAINMENT AND GENERALIZATION OF CONCEPTS EMBODIED SYMBOLICALLY

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Purpose

This investigation was tied closely to the work of Dienes (1960, 1963), and constitutes a contribution to the research conducted under the title of the PARADIGMS Project (Heimer and Lottes, 1973). Its purpose was to examine the effect of instruction employing several non-symbolic embodiments on symbolic operation with, and generalization of, a given concept. Accordingly, the following questions were posed: What effect does achievement to criterion on one, two, or three non-symbolic embodiments of an objective defined over a selected class of variables have on the achievement of a symbolic embodiment of the same objective (i) defined over the same class of variables (symbolic operation with the concept)? (ii) defined over an extended class of variables (primitive generalization of the concept)? (iii) defined over an isomorphic class of variables (mathematical generalization of the concept)?

Research Design and Procedure

These questions were examined over three collections of objectives (θ2, θ3, and θ4) which were designed to teach the concept of equivalent fractions. Each collection contained four content equivalent objectives. The objectives within each collection differed only in the way their given components were embodied. The given component of each of the four objectives in each collection was embodied in one of four ways--as an array of blue and red disks, as an array of shaded and unshaded circles, as a partially shaded rectangle, or as a/b where a and b are both natural numbers and a is less than b.

Three instructional treatments (T1, T2, and T3) were constructed. The first treatment employed the disk embodiment; the second treatment the disk and circle embodiments; the third treatment the disk, circle, and rectangle embodiments. The same set of questions was presented in each treatment in programmed format.

Seventy-eight fifth and sixth grade students were involved in the investigation. After a pretest forty-nine of the students were randomly assigned to the three instructional treatments and received instruction over all three collections. Twenty-nine of the students were randomly assigned to the three instructional treatments and received instruction only over collection θ4. After instruction over each collection the students were given symbolic operation (SO) and primitive generalization (PG) tests of the symbolically embodied objective of that collection. In addition, after instruction over collection θ4 the students were given a mathematical generalization (MG) test.

Findings

It was hypothesized that the proportion of T1 students who reached criterion on the SO (PG, MG) test would be less than the proportion of T2 students who reached criterion on the SO (PG, MG) test which would in turn be less than the proportion of T3 students who reached criterion on the SO (PG, MG) test. All hypotheses were tested at the .15 level of significance. None were supported.

Interpretations

In general, subject to the conditions of the investigation, the results were interpreted to indicate that instruction employing one, two, or three non-symbolic embodiments of a concept had essentially the same effect on symbolic operation with, and generalization of, the concept.

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AN ANALYSIS OF CONTENT STRUCTURE AND COGNITIVE STRUCTURE IN THE CONTEXT OF A PROBABILITY UNIT

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Purpose

During the past decade mathematics curricula have been revised significantly in an effort to provide students with a greater understanding of mathematics. One purpose of the new curricula is to familiarize students with the structure of mathematics. The purpose of this study was to define what is meant by structure in the curriculum and structure in the student's memory following instruction. The study examined the communication of mathematical structure by text to the student.

Rationale

Mathematical structure was defined as the relationships between concepts within a set of abstract systems. Content structure was defined as the web of concepts (words, symbols) and their interrelationships in a body of instructional material. This study investigated three methods--directed graphs, graphs, and task analysis--for mapping the concepts and their interrelations in a programmed text on introductory probability. The theory of directed graphs deals with abstract configurations called digraphs, which consist of points and directed lines. Graph theory differs in that lines are not directed on a graph. Digraph (graph) theory provided a method for transferring written instruction into a structural representation consistent with our definition of mathematical structure. The results of the digraph (graph) analysis were examined with a multidimensional scaling procedure.

Cognitive structure was defined as a hypothetical construct referring to the organization (interrelationships) of concepts in long-term memory. Cognitive structure was investigated using word association and paragraph construction techniques. The word association method consists of presenting the student with a concept in probability and asking him to call forth as many other related mathematical concepts as he can. Responses to the word association test were converted to a median (mean) relatedness coefficient matrix. With the paragraph construction technique, students write a paragraph explaining the mathematical relationship(s) between two concepts. Resulting paragraphs were examined with digraph theory. Both word association and paragraph construction data were analyzed with multidimensional scaling procedures. Content structure and cognitive structure were compared by use of the multidimensional scaling results and Euclidean distance scores.

Research Design and Procedure

Eighty-seven eighth grade students were subjects in the study. The study was conducted during normal school hours and in the regular mathematics classroom. The study lasted approximately two weeks plus a retention test period.

Subjects were divided randomly into experimental and control groups (but not physically separated). All subjects were pretested with attitude toward mathematics, achievement in probability, and word association measures. Following pretesting experimental subjects read the programmed text on probability; control subjects read a programmed text on factors and primes which was not related to probability. No other instruction was given to the subjects. All subjects were posttested with word association, achievement, and paragraph construction instruments. All subjects were given the word association and achievement instruments as retention tests.

Findings

Achievement data were analyzed by a repeated measures analysis of variance. Experimental subjects scored significantly better on the achievement test at post and retention test times as compared to pretest. Experimental subjects scored significantly better on the achievement test at post and retention test times than did control subjects. Word association data were analyzed with a non-parametric analysis of variance procedure. Experimental subjects scored significantly better on the word association test following instruction than did control subjects. Directed graphs and graphs provided a representation of content structure that was interpretable and agreed with the investigator's understanding of the probability material. The word association and paragraph construction techniques were useful in examining the learning of mathematical structure. Experimental subjects cognitive structures resembled the content structure following instruction; this was not true of control subjects. It was concluded that experimental subjects learned how to solve probability problems and learned a significant portion of the text structure as a result of instruction. There was a treatment group difference on achievement, word association, and paragraph construction instruments. The word association and paragraph construction techniques appeared to be useful for formative evaluation and gave qualitatively different information than did the achievement test.

DEVELOPMENTAL ASPECTS OF MATHEMATICAL PROBLEM SOLVING

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Purpose

There is little doubt of the importance of developing an ability to write correct proofs in the study of advanced mathematics. In order to prepare students to study mathematics seriously, it is important that every effort be made to determine the most appropriate places at which to introduce students to the various aspects of proof. Clearly, the psychological processes which are at the base of mathematical proof are as complex and intellectually involved as any in all of human behavior. However, a careful analysis of the simplest aspects of learning to make mathematical proofs can provide a framework for the analysis of more complicated mathematical activities. The purpose of this study was to examine the development of the ability to write a correct mathematical proof by investigating certain developmental aspects of problem solving abilities in a specific mathematical system.

Rationale

Developmental changes in problem solving ability have been difficult to assess. There is considerable evidence that some kinds of problem solving processes are employed by children of all age levels, with the principal difference being in degree of complexity (e.g., Burt, 1919; Estes, 1956; Suppes, 1966). In any problem solving activity, especially those activities identified with proof, logical reasoning plays an important role. However, there is extensive conflicting evidence concerning the development of logical reasoning abilities. According to Piaget (Piaget, 1928; Inhelder and Piaget, 1958) children of ages 11 to 13 years are able to handle certain formal operations (e.g., implication and exclusion) successfully, but they are not able to set up an exhaustive method of proof. This ability to deal with premises that require hypothetico-deductive reasoning is not present until the child is approximately 14-15 years of age.

On the other hand, Hazlitt (1930) has suggested that it may also be the case that young children are unable to demonstrate their ability to produce proofs due to a lack of mathematical experience and sophistication. This conjecture is supported by Hill's (1961) finding that content significantly influences the ability of children to make valid inferences.

The making of mathematical proofs can be considered to be a complex problem solving task. Thus, an analysis of the psychological processes involved in making a proof can be undertaken by investigating several quantitative variables which have been identified as measures of problem solving performance. More specifically, an examination was made of certain developmental aspects of children's problem solving ability in a simple mathematical system. The primary factor under consideration was the relation between problem solving ability and chronological age. The mathematical system used was an altered version of a system devised by Suppes (1965). This system, to be noted by Γ, is concerned with the generation of finite strings if I's and 0's. The single symbol I is the only axiom of Γ and any non-empty finite string of I's and 0's is a well-formed formula of Γ. Five rules of inference are allowed. Rule 1 allows two 0's to be placed at the right end of any non-empty string; Rule 2 allows two I's to be placed at the right end of any non-empty string; Rule 3 allows the removal of an 0 from the right end of a string if and only if it is immediately preceded by I-0, which in turn must be preceded by a non-empty string; Rule 4 allows the removal of an I from the right end of a string if and only if it is immediately preceded by 0-I, which in turn must be preceded by a non-empty string; Rule 5 allows the removal of an I from the left end of a string if there are at least two letters in the string.

Since any non-empty finite string of I's and 0's constitutes a well-formed formula, a theorem of Γ is any well-formed formula that can be obtained from the axiom by a finite number of applications of Rules 1-5. Theorems were limited to strings of from one to five letters.

Research Design and Procedure

In order to investigate the relation between problem solving ability and chronological age (technically, grade level is the independent variable), seventy-six subjects ranging in age from six to eighteen years were randomly selected in the following manner:

A_1: Nineteen subjects from grades 1-3 at a public elementary school.

A_2: Nineteen subjects from grades 4-6 at the same public elementary school as A_1.

A_3: Nineteen subjects from grades 7-9 at a public junior high school.

A_4: Nineteen subjects from grades 10-12 at a public senior high school.

All subjects lived in a middle class area of Columbus, Ohio. Feeder schools were used to insure that all subjects had generally the same socio-economic backgrounds.

Subjects participating in the experiment were asked to supply proofs of theorems of Γ. The tasks (i.e. theorems of Γ) were presented to the subject on a computer terminal using a time-sharing computer-assisted-instruction system. Computer terminals were used in order to control the presentation of tasks so that they were presented to each subject in the same manner. Also, CAI provided the capability for recording and preserving several aspects of subject behavior (e.g., subjects' responses, response times, errors, number of trials).

The subject was confronted with a finite string of I's and O's on the computer terminal. His task was to make the computer type a sequence of strings, the last of which was the original string presented by computer. The subject chose the rule of inference to be applied and pressed the key corresponding to that rule. The computer then applied that rule to the finite string in the last line entered at that point in the proof and printed the resulting string for the subject's inspection.

Seven criterion variables were identified as being appropriate measures of problem solving performance for the given tasks. These variables were: number of theorems proved (abbreviated TP), number of theorems attempted (TA), number of incorrect applications of rules of inference (IAR), number of trials beyond minimal length per theorem proved (TB), trial difficulty per theorem attempted (TD), presolution time per theorem proved (PST), and total time per theorem attempted (TT).

Two one-way multivariate analyses of variance were performed to test the null hypotheses that there are no significant differences in mean performance among groups A_1, A_2, A_3, and A_4 on each of the seven criterion variables. Due to the natural dichotomy of the variables a one-way multivariate analysis of variance OMANOVA) was performed with all non-time variables (TP, TA, IAR, TB) as criteria and a second one-way MANOVA was performed using all time variables (PST, TD, TT) as criteria. Due to the finding of significant multivariate F-tests, a univariate F-test for each variable was performed. Tukey's method of multiple comparisons was used to determine where the differences were.

Findings

Significant differences in mean performance on the problem solving tasks were found among groups when compared on both time variables and non-time variables. Further analysis indicated a significant difference among the groups on four measures of problem solving ability; number of tasks solved, number of incorrect applications of rules of inference, trial difficulty per task attempted, and total time per task attempted. In order to determine which pairs of group means were significantly different on these four variables, Tukey's method of multiple comparisons was used.

Significant results were: $A_1 < A_4$ and $A_1 < A_3$ for TP ($p < .01$); $A_1 > A_4$ ($p < .01$), $A_1 > A_3$ ($p < .01$), and $A_1 > A_2$ ($p < .05$) for IAR; $A_1 > A_4$ and $A_1 > A_3$ for TD $< .01$); and $A_1 > A_4$, $A_1 > A_3$, $A_2 > A_4$, and $A_2 > A_3$ for TT ($p < .01$).

Interpretations

On the basis of the data analysis and with the limitations of the study in mind, it appears that subjects of junior high school age are as capable of solving problems in the experimental mathematical system as subjects of senior high school age. Also, subjects in the upper elementary grades (4-6)are able to successfully solve problems in this system as well as the older subjects except that they require more time.

It is difficult to say what implications these results have for the mathematics classroom. An attempt was made to devise a mathematical system which would be unfamiliar to all subjects. This was done in order to minimize the effect of previous mathematical experiences on the subjects' ability to solve problems. Assuming that the tasks used do involve at least some of the processes involved in mathematical problem solving and mathematical proof, there is reason to believe that even students in the upper elementary grades can be successful at mathematical activities which are closely related to proof. That is, certain aspects of mathematical proof can be understood by children as young as nine years old or younger.

Several ideas for future research are evident. Perhaps the most important suggestion is that future studies should include an examination of subjects' problem solving strategies. Other valuable aspects of problem solving research in this area include an analysis of the correlation between performance on the tasks used in this study and the subject's Piagetian stage of development. Using Piagetian stage of development as a categorization variable in addition to chronological age is another possibility.

The prospects for valuable research in mathematical problem solving using a system like the one used in this study are many and varied. However, it should be emphasized that basic research of this type often has little direct [illegible] ediate implication for the teaching or learning of mathematics. Rather the intent is to help build a theory of mathematical problem solving which will have direct application in the mathematics classroom.